Continuous Time System Identification of Magnetic Bearing Systems Using Frequency Response Data

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Abstract

This paper focuses on identifying nonlinear unstable magnetic bearing systems using a linear model and frequency response data. Subspace system identification methods adopting a Laguerre network in the frequency domain were used to identify continuous time transfer functions for a test-stand magnetic bearing apparatus. The results show that a continuous time multivariable model was successfully identified based on the experimental data.

Keywords: System identification, advanced control system design, magnetic bearing systems, frequency response, subspace methods.

1 Introduction

Magnetic bearings (MBs) have been recognized by industry [1] to have the required attributes, which allow them to be used in applications where traditional bearings and their associated lubrication requirements are impractical. Additionally, as they operate without contact between the rotating shaft and the stationary bearing elements, frictional losses are minimized. But there are some rotational losses due to induced eddy currents. This greatly facilitates the high speed operation of rotating machines, and significantly reduces problems occurring when contact bearings are used.

Typical applications of MBs include flywheel energy storage systems and high speed turbines and compressors. MBs are also used in implanted artificial hearts to solve the blood-clog problem caused by contact bearings. Since no mechanical contact is required to support the rotating pump impeller, problems such as mechanical wear or blood trauma in artificial hearts are now avoided. In addition, a broad range of potential diagnostic information can be attained from position sensing and force generation mechanisms.

However, in spite of all the advantages, MBs are generally open loop unstable systems. This means that an active electronic feedback is required for the bearings to operate stably. The requirement of feedback control (in closed-loop) brings great flexibility into the dynamic response of the bearings. Another obstacle to more widespread commercial and industrial application of MBs is the high sensitivity of the control system to parametric uncertainties and bearing nonlinearities [2]. More often than not a continuous time open-loop model, from controller outputs to sensor outputs, of a MB system is highly desired in order to design advanced control systems aimed at robust stabilization, large operation range and synchronous disturbance attenuation.

System identification has been extensively studied for several decades and has contributed a variety of different identification methods (see for example, [3, 4]). Many papers have been published in the area of identification and control specifically focusing on magnetic bearing systems [5, 6, 7]. All the methods proposed embark on the same goal of improving and enhancing the magnetic bearing systems design by obtaining an improved mathematical model.

The Identification of multi-input multi-output (MIMO) systems of high order is still considered a challenge [8]. These types of systems are usually encountered with modal analysis and the control of flexible structures. However, in most cases where a single model with minimal realization is concerned, state-space models are an excellent choice. As stated by Van Overschee and De Moor [8], the advantage of subspace identification algorithms is that there is no need for an explicit parameterization. The parameter required for user specification is the system order, which can be explicitly determined by inspection of a singular value spectrum. The subspace identification algorithm also requires no nonlinear parametric optimization and no iterative procedures, thus abolishing the problems of local minima and model convergence.

In this paper, an algorithm capable of performing con-
Continuous time model identification is presented. Using the frequency response samples, subspace system identification methods in the frequency domain are used to identify a continuous time transfer function for a test-stand MB system at RMIT University. The test-stand MB system is equipped with PD control systems and two MB actuators which are employed to support a shaft with a disk and two journals are fixed at the mid-point and ends of the shaft.

This paper is organized as follows. First the configuration setup and problem statement of magnetic bearing systems are addressed. The concept of a test-stand is presented, which provides an experimental study of various operating conditions in magnetic bearing systems. Then, the identification algorithm based on the subspace method is presented to model the system. This preliminary attempt started with a single input and single output data system. Then, the numerical result was given to verify the performance capability of the model in identifying the given system.

2 Magnetic Bearing Systems

The development of a test-stand provides such an experimental study of various operating conditions in magnetic bearing systems. The test-stand system available in RMIT University has two bearing actuators. It has four coil currents which need to be controlled, and four shaft displacements which need to be measured. Figure (1) schematically shows the input and output signals in the system where decentralized PD controllers are employed to suspend the shaft. As a result, the open loop model has four inputs and four outputs, leading to a 4x4 transfer function matrix. It is well known that for zero shaft speed, the dynamics in the $x - z$ and $y - z$ planes are decoupled, leading to two separate dynamic systems. Each of the two systems has two inputs and two outputs and, therefore, can be modeled by a 2x2 transfer function matrix.

There are a number of problems that need to be addressed to identify the 2x2 transfer matrix. As the MB actuators are open-loop unstable, a control system needs to be implemented in order to suspend the shaft so as to facilitate data acquisition. It has been noted that a decentralized PD control system is usually sufficient for this purpose. This paper only considers the decoupled sub-system in the $x - z$ plane. The decoupled model in the $y - z$ plane can be investigated in a similar way, and the coupling between the two planes can be further investigated by incorporating the identified sub-system models with gyroscopic effect.

As shown in Figure (1), the shaft displacements are measured at four locations and are represented as $x_L$, $y_L$, $x_R$, and $y_R$ respectively. The air-gap clearance in the MB actuators is 350 µm, leading to a range of [−350,350] µm for these displacements. These displacements are measured by using four proximity sensors from KAMAN Instrument Cooperation, whose gains are adjusted to be 4000 V/m. In addition, there are four pairs of coils in the two bearing actuators along the four radial directions $x_L$, $y_L$, $x_R$, and $y_R$, which are driven by four power amplifiers whose static gains are adjusted to be -0.2 A/V. Currents in these coils are denoted by $i_{xL}$, $i_{yL}$, $i_{xR}$, and $i_{yR}$ respectively.

$$x \in \{x_L, y_L, x_R, y_R\}$$

$$i \in \{i_{xL}, i_{yL}, i_{xR}, i_{yR}\}$$

The force-displacement factor $k_z$ and force-current factor $k_i$ around the equilibrium point ($z=0$; $i=0$) are then calculated to be $k_z=150000$ N/m and $k_i=61$ N/A, respectively. Therefore, the linearized magnetic force, $F_z$, is found to be

$$F_z = k_z x + k_i i$$

In addition, the actuator coil resistance $R_c$ and inductance $L_c$ are measured to be $R_c=602$ and $L_c=30$mH, respectively. The local PD control system is composed of four independent PD controllers, $G_{PDR}(s)$ with

$$G_{PDR}(s) = \frac{-k_{PR}}{1 + \frac{k_{DR} s}{1 + T_{CR}s}}$$

where $k_{PR}=4$, $k_{DR}=7.500 \times 10^{-3}$ and $T_{CR}=1.557 \times 10^{-4}$s. Using the above local PD controllers the shaft and disk in the test-stand MB system are suspended.

Another important issue needing to be addressed in identification of the MB system are the rotor modes (rigid or flexible). Understanding the modes is useful in judging if an identified model is physically reasonable. A detailed discussion of the rigid and flexible modes together with other system dynamics can be found in [9].
3 Continuous Time Subspace Identification Method Using Frequency Response Data

This section introduces a continuous time identification method using frequency response data. Subspace-based identification methods in the frequency domain have been proposed by McKeIvey and colleagues in [10, 11], in addition to the work by De Moor and Vandewalle [12], Verhaegen and Dewilde [13], and Liu et. al [14]. In McKeIvey et al. [11], a bilinear transformation was used in deriving a continuous time state-space model of the form $\frac{1}{s+1}$. In Haverlamp et al. [15], a Laguerre network was proposed in subspace continuous time system identification whereby the scaling factor in the Laguerre network plays a role in the model estimation. This work was further extended by Yang [16] to subspace continuous time using frequency response data. This paper examines the subspace continuous time system identification method using a Laguerre network, in particular with respect to the choice of the scaling factor and the number of terms. This algorithm is applied to the magnetic bearing systems and for completeness, the subspace identification algorithm is also introduced here.

Consider the state-space models of the continuous-time system in the Laplace domain:

$$sX(s) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$ (5)

where $U(s)$, $Y(s)$, $X(s)$ are the Laplace transforms of the system inputs, outputs and state variables respectively, and $A$, $B$, $C$ and $D$ are the system matrices.

The transfer function can be expressed as:

$$G(s) = C(sI_n - A)^{-1}B + D$$ (6)

For a single input and single output system with the input signal being a unit impulse (i.e. $U(s)$ = 1), Equation (5) can be rewritten as:

$$sX(s) = AX(s) + B$$
$$G_m(s) = CX(s) + D$$ (7)

If the model is identified directly using $s = j\omega$, the identification problem becomes ill-conditioned because the condition numbers of the data matrices employed in the identification algorithm increase drastically as the system order increases. To overcome this problem, the $w$-operator corresponds to the all-pass Laguerre filter is introduced as follows:

$$w = \frac{s - p}{s + p}$$
$$s = \frac{1 + w}{1 - w}$$ (8)

where $p > 0$. In the frequency domain, the $w$-operator will become:

$$w(j\omega) = \frac{j\omega - p}{j\omega + p}$$ (9)

The corresponding transfer function and state-space models therefore can be transformed into the following form:

$$G_w(w) = C_w(wI - A_w)^{-1}B_w + D_w$$ (10)

$$wX_w(t) = A_wx_w(t) + B_wu(t)$$ (11)

$$y(t) = C_wx_w(t) + D_wu(t)$$ (12)

where

$$A_w = (A + pI_n)^{-1}(A - pI_n)$$
$$B_w = \sqrt{2p}(A + pI_n)^{-1}B$$
$$C_w = \sqrt{2pC(A + pI_n)^{-1}}$$
$$D_w = D - C(A + pI_n)^{-1}B$$ (13)

and

$$A = p(I_n - A_w)^{-1}(I_n + A_w)$$
$$B = \sqrt{2p}(I_n - A_w)^{-1}B_w$$
$$C = \sqrt{2pC_w(I_n - A_w)^{-1}}$$
$$D = D_w + C_w(I_n - A_w)^{-1}B_w$$ (14)

In frequency domain, equation (7) with the $w$-operator model can be expressed as:

$$w(j\omega)X_w(j\omega) = A_wX_w(j\omega) + B_w$$
$$G_m(j\omega) = C_wX_w(j\omega) + D_w$$ (15)

Before providing the identification algorithm, the data matrices need to be constructed. From $q = 1, \ldots, n$, we multiply the $w$-value to Equation (15) and form the following equation:

$$\begin{bmatrix}
G_m(j\omega) \\
w(j\omega)G_m(j\omega) \\
w^2(j\omega)G_m(j\omega) \\
\vdots \\
w^{n-1}G_m(j\omega)
\end{bmatrix} = \begin{bmatrix}
O_nX_w(j\omega) + \Gamma_n \\
1 \\
w(j\omega) \\
\vdots \\
w^{n-1}(j\omega)
\end{bmatrix}$$ (16)

where

$$O_n = \begin{bmatrix}
C_w \\
C_wA_w \\
C_wA_w^2 \\
\vdots \\
C_wA_w^{n-1}
\end{bmatrix}$$ (17)

$$\Gamma_n = \begin{bmatrix}
D_w \\
C_wB_w \\
C_wA_wB_w \\
\vdots \\
C_wA_w^{n-2}B_w \\
C_wA_w^{n-3} \\
\vdots \\
D_w
\end{bmatrix}$$ (18)

and $n$ is the order of the observability matrix. Given frequency response data for $\omega_k(k = 1, \ldots, M)$ we form the data matrices for identification purposes as shown below:

$$G_{n,M} = O_nX_{wM} + \Gamma_n\Omega_{n,M}$$ (19)
Notice that the operation involved in the above data matrices is actually an adoption of the Laguerre filters that can be expressed as shown below. Details can be found in [17].

\[
P_{n,M} = \begin{bmatrix} \omega^n(j\omega_1)\hat{\Omega}_1 & \ldots & \omega^n(j\omega_M)\hat{\Omega}_M \\ \omega^{n+1}(j\omega_1)\hat{\Omega}_2 & \ldots & \omega^{n+1}(j\omega_M)\hat{\Omega}_M \\ \vdots & \ddots & \vdots \\ \omega^{n+\beta-1}(j\omega_1)\hat{\Omega}_n & \ldots & \omega^{n+\beta-1}(j\omega_M)\hat{\Omega}_M \end{bmatrix} \tag{20}\]

To improve the condition number of the corresponding data matrix, the matrices \(G_{n,M}, X_{w,M}\) and \(\Omega_{n,M}\) are multiplied on the left by \(\sqrt{2p/(j\omega+p)}\).

\[
\hat{G}_{n,M} = \frac{\sqrt{2p}}{j\omega+p} G_{n,M},
\]

\[
\hat{X}_{w,M} = \frac{\sqrt{2p}}{j\omega+p} X_{w,M},
\]

\[
\hat{\Omega}_{n,M} = \sqrt{2p} \Omega_{n,M} \tag{22}\]

Notice that the operation involved in the above data matrices is actually an adoption of the Laguerre filters that can be expressed as shown below. Details can be found in [17].

\[
L_q(j\omega) = \sqrt{2p/(j\omega+p)}^{\beta-1}/(j\omega+p) \tag{24}\]

Since the frequency response data from the magnetic bearing systems contains both measurement noise and the noise from the finite time window in the Fourier transformation [3], an instrumentation variable method is desirable for this application. As in [10], the instrument variable is selected as

\[
P_{n,M} = \begin{bmatrix} w^n(j\omega_1)\hat{\Omega}_1 & \ldots & w^n(j\omega_M)\hat{\Omega}_M \\ w^{n+1}(j\omega_1)\hat{\Omega}_2 & \ldots & w^{n+1}(j\omega_M)\hat{\Omega}_M \\ \vdots & \ddots & \vdots \\ w^{n+\beta-1}(j\omega_1)\hat{\Omega}_n & \ldots & w^{n+\beta-1}(j\omega_M)\hat{\Omega}_M \end{bmatrix} \tag{20}\]

Therefore, equation (19) can be rewritten as

\[
\frac{1}{M} G_{n,M} \Gamma_{\beta,M} = \frac{1}{M} \Omega_{n} X_{w,M} P_{\beta,M}^T + \frac{1}{M} \Gamma_{\Omega,N,M} P_{\beta,M} \tag{26}\]

Introducing the projection matrix, \(\Pi^\perp\) to equation (26) results in zero for the second value on right hand side. Therefore, now the expression goes as:

\[
\frac{1}{M} G_{n,M} \Pi^\perp P_{\beta,M}^T = \frac{1}{M} \Omega_{n} X_{w,M} \Pi^\perp P_{\beta,M}^T \tag{27}\]

where the projection matrix can be defined as

\[
\Pi^\perp = I - \Omega^T (\Omega \Omega^T)^{-1} \Omega \tag{28}\]

On the other hand, the algorithm can also be done by performing the linear quadratic (LQ) factorization and singular value decomposition (SVD) to the working matrix. In this paper the recursive quadratic (RQ) factorization using the modified Gram-Schmidt algorithm is used.

\[
\begin{bmatrix} \hat{G}_{n,M} \\ \hat{P}_{n,M} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \tag{29}\]

Based on the lower triangular square R-matrix, the significant value (\(R_{22}R_{22}^T\)) is executed and therefore used to perform the SVD (\(R_{22}R_{22}^T = USV^T\)). By referring to the singular value of \(S\) after the decomposition, the order of the system can be selected.

The value of \(C_w\) can be directly extracted from the first row of the column vector. To compute the \(A_w\), method called ‘shift-invariance’ can be implemented as mentioned in [12].

\[
\Omega = \begin{bmatrix} C_w \\ C_wA_w \\ C_wA_w^2 \\ \vdots \\ C_wA_w^{n-2} \end{bmatrix}, \quad \theta^\dagger = \begin{bmatrix} C_wA_w \\ C_wA_w^2 \\ \vdots \\ C_wA_w^{n-1} \end{bmatrix} \tag{30}\]

\[A_w = (\Omega^\dagger)^{\dagger}\theta^\dagger \tag{31}\]

where \((\ast)\dagger\) denotes the Moore-Penrose pseudo-inverse.
The \(B_w\) and \(D_w\) can be solved by manipulating Equation (10) using the least squares solution as:

\[
G_m = \Phi \theta \tag{32}\]

\[
\begin{bmatrix} G_m(j\omega_1) \\ \vdots \\ G_m(j\omega_M) \end{bmatrix} = \begin{bmatrix} C_w(j\omega_1)I_n - A_w \end{bmatrix}^{-1} \begin{bmatrix} B_w \\ \vdots \\ B_w \end{bmatrix} \tag{33}\]

\[
\theta = (\Phi^T \Phi)^{-1} \Phi^T G_m \tag{34}\]

### 4 Optimal Selection for Design Parameters

In this algorithm, there are four main variables needing to be determined. They are \(p\)-parameter (adjustable variable for tuning Laguerre filter), \(n\)-parameter (variable for choosing the number of terms for the observability matrix), \(\beta\)-parameter (variable for choosing the number of terms for the instrumental variable matrix) and a variable for determining the system order. The system order is chosen based on observations of the diagonal elements of matrix \(S\) after performing the singular value decomposition (SVD). To perform the identical orientation of the matrix array, the \(\beta\)-value is chosen according to the \(n\)-value. Hence, only two variables are left for observation and these are discussed detail in this section.
Proper scaling of parameter $p$ will lead to efficient model approximation for a large class of linear systems [19]. However, scaling the design parameter for high order unstable nonlinear systems is quite challenging compared to low order systems without noise disturbance. In this paper, we have used a mean square error (MSE) function as a guide for selecting the optimal value of parameters $p$ and $n$.

$$J = \frac{1}{M} \sum_{i=1}^{M} |G(j\omega_i) - \hat{G}(j\omega_i)|^2$$  \hspace{1cm} (35)

With the assumption that a good model will provide a better prediction of the system behaviour, optimal selections of parameters $p$ and $n$ are based on the lowest value of mean square error. Analysis has been done using one of the sample data sets taken from the magnetic bearing systems. Results from the analysis can been seen in Figures (2) and (3) respectively. Based on Figure (2), it can be seen that the model provides a small error when we increase the number of terms for the observability matrix especially after 50. However, increasing the value of $n$ will results in a numerical condition where a division by zero error occurs. From the analysis, the model presents well for $n$-value in the range of 50 to 100.

Results from the analysis of design parameter $p$ can be seen in Figure (3). Based on the observation, the model gives good performance when the value of $p$ is chosen to be in between 80 to 180. Also from this analysis, we have identified that the values of $p$ and $n$ very much depend on the sample data and the order of the system. As a result, an optimal selection of design parameters is necessary in order to make the model reliable for any data samples.

5 Experimental Result

To illustrate the performance of the subspace model in identifying the magnetic bearing systems, the frequency domain data collected from the single input single output (SISO) channel in the closed-loop multi input multi output (MIMO) plant is used. The spectrum analyzer Tektronix 6421 is used for generating an excitation signal and for measuring the input-output data in one of the four channels, while the other three channels are under normal PD control. The Tektronix 6421 spectrum analyzer is able to measure signals up to 100kHz, and is equipped with excellent anti-aliasing filters, which are roughly 2/3 of the
6 Conclusion

This paper has presented an identification approach based on subspace methods with an adopted Laguerre network that produced a continuous-time transfer function model directly from the frequency response data.

This paper is actually the first attempt to implement a subspace method in the novel application of a magnetic bearing systems. Previews based on the early simulation results show that the state-space model could identify the system successfully.

References


