Validity of a single-channel model for a spin-orbit-coupled atomic Fermi gas near Feshbach resonances

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We theoretically investigate a Rashba spin-orbit-coupled Fermi gas near Feshbach resonances, by using mean-field theory and a two-channel model that takes into account explicitly Feshbach molecules in the closed channel. In the absence of spin-orbit coupling, when the channel coupling $g$ between the closed and the open channels is strong, it is widely accepted that the two-channel model is equivalent to a single-channel model that excludes Feshbach molecules. This is the so-called broad resonance limit, which is well satisfied by ultracold atomic Fermi gases of $^6$Li atoms and $^{40}$K atoms in current experiments. Here, with Rashba spin-orbit coupling we find that the condition for equivalence becomes much more stringent. As a result, the single-channel model may already be insufficient to describe properly an atomic Fermi gas of $^{40}$K atoms at a moderate spin-orbit coupling.

We determine a characteristic channel coupling strength $g_c$, as a function of the spin-orbit-coupling strength, above which the single-channel and two-channel models are approximately equivalent. We also find that for narrow resonance with small-channel coupling, the pairing gap and molecular fraction are strongly suppressed by SO coupling. Our results can be readily tested in $^{40}$K atoms by using optical molecular spectroscopy.

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1. INTRODUCTION

As a realization of non-Abelian gauge fields in neutral cold atoms [1–5], spin-orbit (SO)-coupled atomic gases have attracted a lot of attention in recent years. The SO-coupled bosonic gas of $^{87}$Rb atoms was first achieved by Spielman’s group at the National Institute of Standards and Technology in early 2011 [1]. The SO-coupled atomic Fermi gas has also been realized most recently at Shanxi University [4] and at Massachusetts Institute of Technology [5], with $^{40}$K and $^6$Li atoms, respectively. These novel atomic gases have many interesting properties inherent to SO coupling and have potential applications in future quantum technology. A well-known example is the emulation of the long-sought topological superfluids and Majorana fermions [6–8], which lie at the heart of topological quantum information and computation [9,10].

Most previous theoretical studies on SO-coupled Fermi gases are based on a single-channel model [6,11–31]. In this model, the interaction between atoms is described by a single parameter, i.e., the s-wave scattering length $a_s$. The scattering length can experimentally be tuned by using Feshbach resonances. As a result, in the absence of SO coupling, a Fermi gas can cross from a Bose-Einstein condensate (BEC) over to a Bardeen-Cooper-Schrieffer (BCS) superfluid [32], when the scattering length changes from positive to negative values. With SO coupling, the picture of BEC-BCS crossover may be qualitatively altered. For example, for a particular Rashba-type SO coupling, a new two-body bound state—referred to as a rashbon—is formed [11–14]. By increasing the SO coupling strength, the system may change from a BCS superfluid to a BEC of rashbons, even on the BCS side with a negative scattering $a_s < 0$ [12–14]. The pairing gap of this system will be significantly enhanced due to the increased density of state at the Fermi surface [14]. An anisotropic superfluid due to the Rashba SO coupling has also been predicted [13].

A more realistic and complete description of ultracold atomic Fermi gases near Feshbach resonances, however, should be the two-channel model, which includes both atoms in the open channel and Feshbach molecules in the closed channel [33]. In this model, in addition to the background s-wave scattering length $a_{bg}$ between atoms, two other parameters are used in order to fully describe the interaction. These are the detuning energy of Feshbach molecules $v$ and the channel coupling strength $g$ between molecules and Fermi atoms. Therefore, the interaction of the system consists of two parts. The nonresonant part is the contact interaction between atoms, with the strength determined by the background scattering length, while the resonant interaction is induced by the coupling between molecules and atoms. Near Feshbach resonances without SO coupling, it is known that the single-channel and two-channel models are essentially equivalent when the channel coupling strength $g$ is large enough [34,35]. This is the so-called broad resonance condition, satisfied by the Fermi gases of $^{40}$K and $^6$Li atoms, which are so far the two main systems used in the cold-atom laboratory.

In this paper, we aim to examine the equivalence of the single-channel and two-channel models for a Rashba SO-coupled Fermi gas near Feshbach resonances. This is by no means obvious, as fermionic pairing is notably affected by SO coupling at the BEC-BCS crossover. We use mean-field theory and focus on the most interesting resonant limit. Our results show that in the presence of SO coupling, the broad resonance condition is much more difficult to achieve. As a result, for an ultracold atomic Fermi gas of $^{40}$K atoms, which is known to be well described by the single-channel model without SO coupling, we may have to use a two-channel model already at a moderate SO coupling strength.
Our paper is organized as follows. In Sec. II, we introduce the model Hamiltonian. In Sec. III, we diagonalize the Hamiltonian using mean-field theory to obtain the grand thermodynamic potential and solve the resulting coupled mean-field equations. In Sec. IV, we discuss the equivalence between the single-channel and the two-channel models. In Sec. V, we show how to test experimentally the difference between the two models, by using optical molecular spectroscopy [36]. Finally, we summarize in Sec. VI.

II. MODEL HAMILTONIAN

We consider a three-dimensional (3D) resonantly interacting atomic Fermi gas with Rashba-type SO coupling, described by the two-channel model Hamiltonian,

$$\mathcal{H} = \mathcal{H}_{SO} + \mathcal{H}_m + \mathcal{H}_I,$$  

where $\mathcal{H}_{SO}$ and $\mathcal{H}_m$ stand for the noninteracting Hamiltonian of SO-coupled atoms in the open channel and of Feshbach molecules in the closed channel, respectively. The interaction Hamiltonian $\mathcal{H}_I = \mathcal{H}_{am} + \mathcal{H}_{as}$ includes both the atom-molecule coupling between the two channels ($\mathcal{H}_{am}$) and the background interaction between open-channel atoms ($\mathcal{H}_{as}$).

For atoms, we take the single-particle Rashba SO Hamiltonian

$$\mathcal{H}_{SO} = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 \lambda}{2m} k_x \cdot \sigma, \tag{2}$$

where $k_\perp \equiv (k_x, k_y)$ and $\sigma_\perp \equiv (\sigma_x, \sigma_y)$ are, respectively, the in-plane momentum and in-plane Pauli matrix, and $\lambda$ is the Rashba SO coupling strength. Note that the standard representation of the Rashba SO coupling is given by $\lambda(k, \sigma_z - k_x \sigma_y)$ [13]. Here, for convenience we have performed a spin rotation to rewrite the Rashba term in a slightly different but fully equivalent form, $\lambda(k, \sigma_z + k_x \sigma_y)$ [14]. In the second quantized form,

$$\mathcal{H}_{SO} = \sum_{k, \sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + \frac{\hbar^2 \lambda k^2}{2m} (e^{-i\varphi_k} a_{k\uparrow}^\dagger a_{k\downarrow} + \text{H.c.}), \tag{3}$$

where $a_{k\sigma}^\dagger$ is the creation operator for atoms with momentum $k$ in spin state $\sigma$, $\epsilon_k \equiv \hbar^2 k^2/(2m)$, and $\varphi_k \equiv \arg(k_x + i k_y)$. To diagonalize this single-particle Hamiltonian, we introduce the field operators in the helicity basis labeled “±”:

$$\begin{pmatrix} h_{k\uparrow} \\ h_{k\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i \varphi_k} \\ e^{i \varphi_k} & -1 \end{pmatrix} \begin{pmatrix} a_{k\uparrow}^\dagger \\ a_{k\downarrow}^\dagger \end{pmatrix},$$

with which the single-particle Rashba Hamiltonian becomes diagonal,

$$\mathcal{H}_{SO} = \sum_{k} (\epsilon_k + h_{k\uparrow} - \epsilon_k - h_{k\downarrow}) a_{k\uparrow}^\dagger a_{k\downarrow}. \tag{5}$$

Note that, in the helicity basis, the single-particle dispersion relation now breaks into two branches: $\epsilon_{k\uparrow} = \epsilon_k + \hbar^2 \lambda k_x^2/(2m)$ for the upper branch and $\epsilon_{k\downarrow} = \epsilon_k - \hbar^2 \lambda k_x^2/(2m)$ for the lower branch. To describe Feshbach molecules, we use annihilation operators $b_q$. The energy of molecules is denoted $2\nu$, which, after renormalization [33], is related to the detuning energy from the threshold of Feshbach resonance $B_0$, i.e., $2\nu_0 = \Delta \mu(B - B_0)$, where $\Delta \mu \equiv 2\mu_a - \mu_m$ is the magnetic moment difference between the atomic ($2\mu_a$) and the bound molecular state ($\mu_m$) [33]. The Hamiltonian of Feshbach molecules may be written as

$$\mathcal{H}_m = 2\nu \sum_q b_q^\dagger b_q. \tag{6}$$

Finally, the interaction Hamiltonian is given by $\mathcal{H}_I = \mathcal{H}_{aa} + \mathcal{H}_{am}$, where

$$\mathcal{H}_{aa} = U_{bg} \sum_{kkq} a_{q/2+k\uparrow}^\dagger a_{q/2-k\downarrow}^\dagger a_{q/2-k\downarrow} a_{q/2+k\uparrow} \tag{7}$$

is the nonresonant interaction between atoms, with the strength given by the background s-wave scattering length after renormalization [33], $U_{bg} = 4\pi \hbar^2 a_{bg}/m$, and

$$\mathcal{H}_{am} = g \sum_{kq} [b_{q/2+k\uparrow}^\dagger a_{q/2-k\downarrow}^\dagger a_{q/2-k\downarrow} + \text{H.c.}] \tag{8}$$

is the resonant interaction between atoms and molecules, with the strength parameterized by $g$. After renormalization, the magnitude of the channel coupling strength $g$ is related to the width of the Feshbach resonance $W$, i.e., $g \equiv \sqrt{\Delta \mu W U_{bg}}$. We note that, in the two-channel model, one may define an effective s-wave length [35],

$$a_s = a_{bg} \left(1 - \frac{W}{B - B_0}\right) = a_{bg} - \frac{g^2 m}{2\nu_0^2} \frac{m}{4\pi \hbar^2}. \tag{9}$$

III. MEAN-FIELD THEORY

We use the standard mean-field theory to solve the two-channel model, Eq. (1), by assuming that all the molecules and Cooper pairs condense into the zero-momentum state. Thus, we set $q = 0$ in the interaction Hamiltonian $\mathcal{H}_I$. Here, we have excluded the possibility of an inhomogeneous superfluid phase (i.e., $q \neq 0$), which may exist in the presence of an in-plane Zeeman field [37]. This is consistent with the two-body calculation [13,27,38] that the ground state of two particles in our Hamiltonian always has zero center-of-mass momentum. Following the procedure in Ref. [33], we introduce the following field parameters:

$$\phi_m = (b_0), \tag{10}$$

$$p = \sum_k (a_{k\uparrow}^\dagger a_{-k\downarrow}), \tag{11}$$

$$f = \sum_k (a_{k\uparrow}^\dagger a_{k\uparrow}) = \sum_k (a_{k\downarrow}^\dagger a_{k\downarrow}), \tag{12}$$

where $\phi_m$ is the molecular field in the closed channel, $p$ is the pairing field, and $f$ is half of the number of fermionic atoms in the open channel. The interaction Hamiltonian $\mathcal{H}_{aa}$ can therefore be written as

$$\mathcal{H}_{aa} \sim \frac{U_{bg}}{2} \sum_{kk} f a_{k\sigma}^\dagger a_{-k\sigma}^\dagger - \sum_{k} (p a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \text{H.c.}) - |p|^2 - f^2. \tag{13}$$

Similarly, we approximate $\mathcal{H}_{am}$ as

$$\mathcal{H}_{am} \sim -g \sum_k (\phi_m a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \text{H.c.}). \tag{14}$$
Thus, within the mean field the total Hamiltonian is given by
\[ \mathcal{H} = -U_{bg}(|p|^2 + f^2) + \sum_{k} \epsilon_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k \sigma} U_{bg} f \hat{a}_{k \sigma}^\dagger \hat{a}_{k \sigma} + 2v|\phi_m|^2 - \sum_{k} \left( (U_{bg} p + g\phi_m) \hat{a}_k^\dagger \hat{a}_{k \sigma} + \text{H.c.} \right). \] (15)

where \( \tau \equiv \pm \) is the index of the helicity branch. By defining an order parameter \( \Delta = -(U_{bg} p + g\phi_m) \) and rewriting all the field operators in the helicity basis, the total mean-field Hamiltonian becomes
\[ \mathcal{H} = -U_{bg}(|p|^2 + f^2) + 2v|\phi_m|^2 + \sum_{k} (\epsilon_k + U_{bg} f) \hat{h}_k^\dagger \hat{h}_k - \Delta \sum_{k} \left[ e^{-i\phi_k} \hat{h}_{k \uparrow}^\dagger \hat{h}_{k \downarrow} + e^{i\phi_k} \hat{h}_{k \downarrow}^\dagger \hat{h}_{k \uparrow} + \text{H.c.} \right]. \] (16)

To determine the variational field parameters \( \phi_m, p, \) and \( f \), we diagonalize \( \mathcal{K} = \mathcal{H} - \mu N \) by using Bogoliubov transformation and calculate the grand thermodynamic potential \( \Omega \). Here, \( N \equiv \sum_{k} \alpha_k^\dagger \alpha_k + 2 \sum \beta_k^\dagger \beta_k \) is the operator of the total number of atoms and \( \mu \) is the chemical potential. Using the field operators for Bogoliubov quasiparticles \([33], \alpha_k^+ \) and \( \alpha_k^- \), \( \mathcal{K} \) takes the diagonal form,
\[ \mathcal{K} = \sum_{k \sigma} E_{k \sigma} \alpha_k^\dagger \alpha_k - U_{bg}(|p|^2 + f^2) + 2v|\phi_m|^2 + \sum_{k} \left[ (\xi_k + U_{bg} f) - \frac{E_{k \uparrow} + E_{k \downarrow}}{2} \right]. \] (17)

where \( \xi_k \equiv \epsilon_k - \mu \) and the energies of Bogoliubov quasiparticles \( E_{k \sigma} \) are given by
\[ E_{k \sigma} = \sqrt{\xi_k^2 + \frac{\hbar^2 k_{\sigma}^2}{2m} + U_{bg} f^2} + |\Delta|^2. \] (18)

At temperature \( T \), it is straightforward to write down the grand thermodynamic potential,
\[ \Omega = \sum_{k} \left[ \xi_k + U_{bg} f - \frac{E_{k \uparrow} + E_{k \downarrow}}{2} - U_{bg}(|p|^2 + f^2) + 2v|\phi_m|^2 - k_B T \sum_{k \sigma} \ln \left[ 1 + e^{-\xi_k/k_B T} \right] \right]. \] (19)

The field parameters \( \phi_m, p, \) and \( f \) must satisfy the coupled self-consistent equations, \( \partial \mathcal{L}/\partial f = 0 \), \( \partial \mathcal{L}/\partial p = 0 \), and \( \partial \mathcal{L}/\partial \phi_m = 0 \). Furthermore, the chemical potential is determined by the total number of atoms \( N \), i.e.,
\[ N = -\frac{\partial \Omega}{\partial \mu} = 2f + 2 \phi_m^2. \] (20)

These four coupled equations can be solved to obtain the pairing order parameter \( \Delta = -(U_{bg} p + g\phi_m) \) and chemical potential \( \mu \).

**IV. RESULTS AND DISCUSSION**

To clearly contrast the two-channel model with single-channel model, we focus on the resonant limit and neglect the background interaction. By setting \( U_{bg} = 0 \) (and therefore \( \Delta = -g\phi_m \)) and renormalizing the energy of molecules \( v \) by using \([33]\), we obtain the coupled gap equation and number equation in the two-channel model \( \tau \equiv \pm \),
\[ \frac{2(\nu_0 - \mu)}{g^2} = \sum_{k} \left[ \sum \frac{1}{2} - n_{k \sigma} \right] - \frac{1}{2 \epsilon_{k \sigma}} \] (22)
\[ N = \frac{2 \Delta^2}{g^2} = \sum_k \left[ 1 - \sum \left( \frac{1}{2} - n_{k \sigma} \right) \right] \frac{\epsilon_{k \sigma} - \mu}{E_{k \sigma}} \] (23)

where \( n_{k \pm} \equiv 1/(e^{\epsilon_{k \pm}/k_B T} + 1) \) is the Fermi-Dirac distribution function. In contrast, the gap and number equations in the single-channel model are given by \([13]\),
\[ -\frac{m}{4 \pi \hbar^2 a_s} = \sum_{k} \left[ \sum \frac{1}{2} - n_{k \sigma} \right] - \frac{1}{2 \epsilon_{k \sigma}} \] (24)
\[ N = \sum_k \left[ 1 - \sum \left( \frac{1}{2} - n_{k \sigma} \right) \right] \frac{\epsilon_{k \sigma} - \mu}{E_{k \sigma}} \] (25)

respectively. By recalling from Eq. (9) that the effective \( s \)-wave scattering length in the two-channel model is \(-4\pi \hbar^2 a_s/m = \frac{g^2}{(2\nu_0)}\), it is clear that the gap and number equations in both models have the same structure. However, additional terms, \(-2\mu/g^2\) and \(-2\Delta^2/g^2\), appear in the two-channel gap and number equations, respectively. For a finite SO coupling constant \( \lambda \), if \( g \to \infty \), \( 2\mu/g^2 \) and \( \Delta^2/g^2 \) go to 0. Then the equations of the two models become exactly the same. This is the same as the situation without SO coupling. In other words, the single-channel model and two-channel model coincide with each other in the broad resonant limit, as they should. However, for a finite channel coupling strength \( g \), if \( \lambda \) is sufficiently large, a deep two-body bound state (i.e., rashbon) appears, with a divergent chemical potential \( |\mu| \to -\infty \); see Fig. 1(b). Thus, we cannot neglect \( 2\mu/g^2 \) in Eq. (22) anymore and the two models are no longer equivalent. In this strong SO coupling limit, we anticipate a qualitative difference between the single-channel and the two-channel models.

Let us turn to detailed numerical calculations. For simplicity, we consider the case in which the temperature is 0 and the system is exactly at Feshbach resonance \((\nu_0 = 0 \) and \( a_s^{-1} = 0)\). To characterize the width of Feshbach resonances, we introduce a dimensionless channel coupling constant,
\[ g_0 = \frac{2m}{\hbar^2 k_F^2 g}, \] (26)
where \( k_F = (3\pi^2 N/V)^{1/3} \) is the Fermi wavelength. We take the energy and length in the units of the Fermi energy \( E_F = \hbar^2 k_F^2/2m \), and the inverse Fermi wavelength \( k_F^{-1} \), respectively. The SO coupling strength is measured by the dimensionless parameter \( \lambda/k_F \). In experiments, the typical magnitude of SO coupling strength is at the order of the Fermi wavelength, i.e., \( \lambda = O(k_F) \).

Figure 1 reports the evolution of the pairing gap [Fig. 1(a)] and chemical potential [Fig. 1(b)] with decreasing resonance width. For comparison, the prediction of the single-channel model is also shown by solid lines. As shown in Fig. 1(a), for
systems with a low SO coupling strength, the single-channel model and two-channel model give the same results when $g_0$ is large enough (i.e., $g_0 > 20$). In the case of $^6$Li atoms with $g_0 \approx 600$ [(purple) dotted line in Fig. 1], we cannot see the difference from the single-channel prediction. This means that the broad resonance condition is always valid for $^6$Li atoms. However, for a smaller $g_0$, e.g., $^4$K atoms with $g_0 \approx 50$, the pairing gap deviates clearly from the single-channel prediction at the typical experimental SO coupling strength $\lambda/k_F = 3$, although the two models give essentially the same pairing gap in the absence of SO coupling. With increasing SO coupling strength, the difference between the two models becomes more significant. For even smaller $g_0$ (i.e., $g_0 = 5$), it is interesting that the dependence of the pairing gap on the SO coupling strength changes qualitatively. The pairing gap starts to decrease with increasing SO coupling strength and vanishes at a sufficiently large SO coupling.

This dramatic change is somehow not anticipated, as the pairing gap is always enhanced by SO coupling in the single-channel model. It is closely related to the anisotropic superfluidity caused by the Rashba SO coupling. As discussed in Ref. [13], due to SO coupling the fermionic superfluid has mixed singlet and triplet components. The fraction of triplet pairing grows with increasing SO coupling strength. Thus, within the single-channel model, the amplitude of the pairing gap reflects both singlet and triplet pairing strengths and increases as the SO coupling increases. In the two-channel model, however, the most important resonant-interaction Hamiltonian $\mathcal{H}_{\text{int}}$ is of $s$-wave character and hence favors the singlet pairing. As the triplet pairing is favored by Rashba SO coupling, the resonance width and SO coupling have opposite effects on the pairing gap and destruct with each other. The destruction becomes very pronounced with decreasing resonance width, leading to a completely suppressed pairing gap at a large SO coupling and narrow resonance width.

The suppression of the pairing gap can also be mathematically understood from the two-channel gap equation, Eq. (22), where we may treat $2(\nu_0 - \mu)$ as the effective energy detuning of Feshbach molecules. With increasing SO coupling, the chemical potential will diverge to $-\infty$, and hence the effective energy detuning is pushed up to the BCS limit. As a result, the pairing gap becomes significantly suppressed.

Now, it is clear that the broad resonance limit becomes much more difficult to reach in the presence of Rashba SO coupling.
To quantitatively characterize the broad resonance condition, we show in Fig. 2(a) the behavior of the pairing gap $\Delta$ as a function of the resonance width $g_0$, for some selected values of $\lambda/k_F$. As $g_0$ increases, the pairing gap $\Delta$ grows rapidly at first, then saturates to the prediction of the single-channel model. Quantitatively, we may define a critical $g_c$ in such a way that, above $g_c$, the relative difference in the pairing gaps predicted by the two models is less than 5%. Figure 2(b) presents $g_c$ as a function of SO coupling strength. It gives a qualitative phase diagram. Above $g_c$, we may safely use the single-channel model to describe the Rashba SO-coupled atomic Fermi gas near Feshbach resonances. However, below $g_c$, the two-channel model must be adopted. For $^{40}$K atoms with $g_0 \simeq 50$, we find that the single-channel model becomes insufficient at a moderate Rashba SO coupling, $\lambda \sim 3k_F$.

V. EXPERIMENTAL RELEVANCE

To experimentally test our predictions, we consider optical molecular spectroscopy, which projects Feshbach molecules onto a vibrational level of an excited molecule. The rate of excitations enables precise measurement of the fraction of the closed-channel Feshbach molecules in the paired state, although the fraction can be extremely small [36]. Near resonance, the paired state may be treated as dressed molecules [35,36],

$$|\text{dressed}\rangle = e^{i\phi_m} \sqrt{1 - Z_m}|\text{open}\rangle + \sqrt{Z_m}|\text{closed}\rangle,$$

where $Z_m$ can be identified as the component fraction of Feshbach molecules, i.e., $Z_m = 2|\phi_m|^2/N$. As a concrete example, in Fig. 3 we show the fraction for $^{40}$K atoms (with $g_0 \simeq 50$) as a function of the SO coupling strength. As the SO coupling increases, the population of Feshbach molecules is almost flat at first. However, after the coupling reaches a critical value, $\lambda \simeq k_F$, it grows very rapidly.

The impact of SO coupling on the population of Feshbach molecules is best seen in Fig. 4, where we present the fraction as a function of the resonance width at several SO coupling strengths. For a given nonzero SO coupling, the fraction is a nonmonotonic function of the resonance width. Upon increasing $g_0$ from the broad resonance limit, the fraction first grows, then drops to zero, as a result of the competition between SO coupling and resonance width, as mentioned earlier. In the limit of narrow resonance, the vanishing molecular fraction is consistent with the suppression in the pairing gap at a large SO coupling, as shown in Fig. 1(a), due to the relation $\Delta = -g\phi_m$. In contrast, in the absence of SO coupling the molecular fraction increases steadily with decreasing resonance width. We may define a characteristic $g_{\text{max}}$ at which the fraction reaches its peak value. As shown in the inset in Fig. 4, when $\lambda/k_F = 0$, namely, there is no SO coupling, $g_{\text{max}} = 0$, and the population reaches unity in the limit of $g_0 = 0$. As the SO coupling increases, $g_{\text{max}}$ increases. We emphasize that for a small resonance width $g_0$, even a small SO coupling could lead to a strong suppression of the population of Feshbach molecules.

VI. SUMMARY

In conclusion, we have investigated a Rashba SO-coupled Fermi gas near Feshbach resonances, using a two-channel model. When the SO coupling strength is low and the Feshbach resonance is broad, the two-channel model is equivalent to the single-channel model, as we might anticipate [34,35]. However, for a given resonance width, if the SO coupling strength is sufficiently high, these two models are no longer equivalent. Moreover, for a narrow resonance the pairing gap and the fraction of Feshbach molecules are strongly suppressed by SO coupling. We could test these predictions by measuring experimentally the molecular fraction using optical molecular spectroscopy [36]. We have characterized quantitatively the equivalence of the two models by introducing a critical resonance width, above which the two models are approximately the same. By calculating the dependence of the critical resonance width on the SO coupling strength, we have found that the single-channel model may break down for Rashba SO-coupled $^{40}$K atoms at a moderate SO coupling strength.

Our results are obtained within the mean-field theory, which is known to provide a qualitative picture of resonantly interacting atomic Fermi gases. For quantitative purposes, the crucial pairing fluctuation must be included. This may
be addressed by using many-body $T$-matrix theories in the future [35,39,40].

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