Abstract—This paper concerns the adaptive fast finite time control of a class of nonlinear uncertain systems of which the upper bounds of the system uncertainties are unknown. By using the fast non-smooth control Lyapunov function and the method of so-called adding a power integrator merging with adaptive technique, a recursive design procedure is provided, which guarantees the fast finite time stability of the closed-loop system. It is proved that the control input is bounded, and a simulation example is given to illustrate the effectiveness of the theoretical results.

I. INTRODUCTION

A finite time stabilization problem was initially studied in the literature of optimal control [1]. The finite time control laws [2-8] are usually time-varying, discontinuous, or even depending directly on the initial conditions of considered systems [9]. In [10], a Lyapunov stability theorem has been used to test the finite time stability of continuous nonlinear system. In [11, 12], the idea of non-smooth control was used to test the finite time stability of continuous nonlinear systems [9]. In [10], a Lyapunov stability theorem has been proposed and studied to solve the problem stated above. We first define the appropriate unknown parameters, and then successfully solve the mentioned problem using the fast non-smooth control Lyapunov function and the adding a power integrator approach [4, 14, 15], merging with adaptive technique. Finally, we will show that under some weak conditions, the control input is bounded.

II. PROBLEM FORMULATION

Consider the $n^{\text{th}}$ order SISO nonlinear system with mismatched uncertainties:

$$\dot{x}_i = x_{i,u} + \Delta_i (x, t, u), \quad 1 \leq i \leq n-1,$$

$$\dot{x}_n = u(t) + f(x, t) + \Delta_n (x, t, u) + d_n (t, x), \quad (1)$$

where $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ is a vector of measurable states, $u(t) \in \mathbb{R}$ represents the input signal. The functions $\Delta_i (x, t, u) = 0$ for all $x, t, u$. For this problem, the Bernoulli-Jacobi equation [13] like fast non-smooth control Lyapunov function has been used in [7] to design a continuous terminal sliding mode control law. This fast non-smooth control Lyapunov function can deliver faster finite time stability of the closed-loop system that cannot be realized by using either control Lyapunov function alone.

It is observed that in most of the existing works mentioned, the upper bounds of the system uncertainties are assumed known or unknown but upper bounded by known positive constants (or smooth functions). Thus, naturally an interesting question arises: Without these assumptions, how to design a robust finite time controller? This question motivates us to investigate the problem of adaptive control design that can guarantee the finite time stability of the closed-loop system by means of continuous feedback.

In this paper, an adaptive fast finite time control scheme is proposed and studied to solve the problem stated above. We first define the appropriate unknown parameters, and then successfully solve the mentioned problem using the fast non-smooth control Lyapunov function and the adding a power integrator approach [4, 14, 15], merging with adaptive technique. Finally, we will show that under some weak conditions, the control input is bounded.
(1) $V(x)$ is positive definite;

(2) $\dot{V}(x) + \alpha V(x) + \beta \sigma' \gamma(x) \leq 0$.

Then, the origin of system (4) is globally finite time stable and the settling time, depending on the initial state $x(0) = x_0$, is given by

$$T(x_0) \leq \frac{1}{\alpha(1-\gamma)} \ln \left( \frac{\alpha V(x_0)}{\beta} \right).$$

Lemma 1 (Lin and Qian [16]): Let $c$, $d$ be positive real numbers and $\phi(x)$, $\varphi(x)$, and $\theta(x)$ be real-valued continuous functions. For any continuous function $\tau(x) > 0$, there is a continuous function $\rho(x) > 0$ such that

$$\left| \phi(x) \right|^b + \left| \varphi(x) \right|^b \leq \tau(x) \left( \phi(x) \right)^{\frac{b}{2}} + \left( \varphi(x) \right)^{\frac{b}{2}} \rho(x).$$

Lemma 2 (Huang, Lin, and Yang [4]): For any real numbers $x_i$, $i = 1, 2, \ldots, n$ and $0 < \eta \leq 1$, the following inequality holds:

$$\left( x_1 + \cdots + x_n \right)^{\frac{1}{\eta}} \leq \left( x_1^\eta + \cdots + x_n^\eta \right)^{\frac{1}{\eta}}.$$  

When $b = p / q \leq 1$, where $p > 0$ and $q > 0$, odd integers,

$$\left| x^b - y^b \right| \leq 2^{-\frac{1}{2}} \left| x - y \right|^\frac{b}{2}.$$  

Proof: The proof of inequality (8) can be found in [17]. For inequality (7), let $q = 1 / \eta$. Then, by the Taylor expansion formula with the integration remainder, we have

$$\left( x_1 + \cdots + x_n \right)^{\eta} \geq \left( x_1^{\eta} + \cdots + x_n^{\eta} \right).$$

(9)

that imply

$$\left( x_1^{\eta} + \cdots + x_n^{\eta} \right) \geq \left( x_1 + \cdots + x_n \right)^{\frac{1}{\eta}},$$

which is identical to (7).

III. ADAPTIVE FAST FINITE TIME CONTROL

In this section, we propose a multiple-surface sliding mode [18-21] like procedure to construct adaptive fast finite time controllers of system (1). The controllers are designed in $n$ steps:

Step 1: Define $n$ sliding mode surfaces $s_i = x_i^{p_i} - x_i^{q_i}$ for $i = 1, \ldots, n$, where $x_i$ represents the desired value of state $x_i$, $p_i$ and $q_i$ are odd positive integers, $q_i \geq p_i$, $0 < p_i / q_i < \ldots < p_i / q_i \leq 1$. Let $Q_i = p_i / q_i$, $x_i = x_i$, $\hat{\theta}_i = \theta - \hat{\theta}_i$ where $\hat{\theta}_i$ is the estimate of $\theta$. Let us first consider subsystem

$$x_i = x_i + \Delta_i(x, t, u),$$

where $\Delta_i(x, t, u) \leq \left| x_i \right| \sigma_i(x_i)$. In the case that parameter $\sigma_i(x_i)$ is unknown, we propose a robust adaptive virtual control law to guarantee the subsystem finite time stable as detailed in Lemma 3.

Lemma 3: Consider subsystem (11), if $s_i = 0$ and the virtual controller $x_i$, is chosen as

$$x_i = -\frac{1}{\alpha_i} \dot{x}_i + \frac{2}{\beta_i} \sigma_i(x_i) \left( \dot{x}_i \right)^{-\gamma},$$

where $\alpha_i = \frac{1}{\alpha_i}$, $\beta_i = \frac{1}{\beta_i}$, $0 < \gamma < 1$, and the parameter $\hat{\theta}_i$ is updated by

$$\dot{\hat{\theta}}_i = \frac{1}{\alpha_i} \frac{1}{\beta_i} \sigma_i(x_i) \left( \dot{x}_i \right)^{-\gamma}.$$  

then, for any finite initial condition $x_i(0)$ and $\hat{\theta}_i(0)$, the system state $x_i$ will converge to zero in finite time

$$T_i \leq \frac{1}{\alpha_i} \frac{1}{\beta_i} \ln \left( \frac{\alpha_i V(x_i(0)) + \beta_i}{\beta_i} \right).$$

where $V_i(x_i(0))$ is the Lyapunov function candidate (15).

Proof: To show that $x_i$ converges to zero in finite time (14), based on Theorem 1, we first find a Lyapunov function candidate $V_i(x_i) > 0$ such that $\dot{V}_i(x_i) + \alpha_i \sigma_i(x_i) + \beta_i \sigma_i(x_i) \leq 0$, $\forall x_i$. Let us consider the following Lyapunov function candidate:

$$\dot{V}_i(x_i) = \frac{1}{1 + \alpha_i} \frac{1}{\beta_i} \sigma_i(x_i).$$

(15)

Since $\frac{1}{2} \tilde{\theta}_i^2 \geq 0$, we have

$$\dot{V}_i(x_i) \leq \frac{1}{1 + \alpha_i} \frac{1}{\beta_i} \sigma_i(x_i) + \frac{1}{2} \tilde{\theta}_i^2.$$  

(16)

Differentiating (16) with respect to time, we have

$$\dot{V}_i(x_i) \leq s_i \left( x_i + \Delta_i(x, t, u) - \tilde{\theta}_i \right)$$

$$\leq s_i \left( x_i + \frac{1}{\beta_i} \sigma_i(x_i) - \hat{\theta}_i \right)$$

$$= s_i \left( x_i - x_i \right) + s_i x_i + s_i \dot{x}_i + s_i x_i$$

$$+ s_i \sigma_i(x_i) - \tilde{\theta}_i \right) \sigma_i(x_i) - \hat{\theta}_i \right) \sigma_i(x_i).$$

(16)

Letting $\theta = \left| x_i \right| \sigma_i(x_i)$, we get

$$\dot{V}_i(x_i) \leq s_i \left( x_i - x_i \right) + s_i x_i$$

$$+ s_i \sigma_i(x_i) - \tilde{\theta}_i \right) \sigma_i(x_i) - \hat{\theta}_i \right) \sigma_i(x_i).$$

(16)

Substituting the virtual control (12) and updating law (13) into (18), if $s_i = 0$, we have

$$\dot{V}_i(x_i) \leq -2 \beta_i \sigma_i(x_i) - 2 \beta_i \sigma_i(x_i)$$

$$\leq -\beta_i V_i - \alpha_i V_i$$

(19)
which means that \( x_i \) converges to zero in finite time (14).

Step \( i \) \((2 \leq i \leq n-1)\): Similar procedures are taken for each steps when \( i = 2, \ldots, n-1 \) as in Step 1. Let us consider subsystems

\[
\dot{x}_i = x_{i+1} + \Delta_i \left( x_{1}, \ldots, x_i, t, u \right).
\]

The sufficient condition of the existence of adaptive virtual control laws to render (20) finite time stable are detailed in Corollary 1. Following the design steps in Step 1 and using Lemmas 1–2, we are able to prove this corollary.

**Corollary 1**: Consider \( i \) subsystems (20) where \( i = 2, \ldots, n-1 \), if \( s_{i+1} = 0 \) and there exist a set of \( C^i \) Lyapunov functions \( V_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) > 0 \), and a set of \( C^0 \) virtual controller \( x_{i+1}, \ldots, x_{n} \) defined by

\[
x_{i+1} = -x_{i}^{1(\gamma+\beta)} \kappa_i(x_{1}, \ldots, x_i, \hat{t}_i),
\]

\[
\dot{\hat{t}}_i = x_{i+1}^{1(\gamma+\beta)},
\]

with, \( \kappa_i(x_{1}, \ldots, x_i, \hat{t}_i) > 0, \ldots, \kappa_i(x_{1}, \ldots, \hat{t}_i) > 0 \) being \( C^i \), such that

\[
V_i(x_{1}, \ldots, x_i, \hat{t}_i) = -\alpha_i V_i(x_{1}, \ldots, x_i, \hat{t}_i) - \beta_i V_i(x_{1}, \ldots, x_i, \hat{t}_i) + \left( n-i+1 \right) \left( x_{i}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{i}^{1(\gamma+\beta)} + s_{i}(x_{i+1} - x_{i+1})
\]

(22a)

(22b)

where \( \alpha_i > \ldots > \alpha > 0, \beta_i > \ldots > \beta > 0 \), then for any finite initial condition \( x_{1}(0), \hat{t}_i(0) \), the system state \( x_i \) will converge to zero in finite time

\[
T_i \leq \frac{1}{\alpha_i}(1 - \gamma) \ln \left( \frac{\alpha_i^{1(\gamma+\beta)} (x_0(0)) + \beta_i}{\beta_i} \right).
\]

**Proof**: We prove this corollary by induction. The Lyapunov function candidate

\[
V_{i}(\cdot) = V_{i-1}(\cdot) + W_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i),
\]

(24)

where

\[
W_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) = \int_{x_i} \left( x_i^{1(\gamma+\beta)} - x_i^{1(\gamma+\beta)} \right) dx.
\]

(25)

The Lyapunov function \( V_i(\cdot) \) thus defined has several useful properties collected in the following two propositions.

**Proposition 1**: \( W_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) \) is \( C^i \). Moreover,

\[
\frac{dW}{dx_i} = x_i^{1(\gamma+\beta)} - x_i^{1(\gamma+\beta)} = s_i,
\]

\[
\frac{dW}{dx_j} = (x_j - x_{i+1}) \frac{d(-x_j^{1(\gamma+\beta)})}{dx_j},
\]

(26)

**Proof**: The proof is straightforward and therefore is omitted.

**Proposition 2**: \( V_{i-1}(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) \) is \( C^i \), positive definite and proper satisfying

\[
V_{i-1}(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) \leq \left( n-i+1 \right) \left( x_{i}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{i}^{1(\gamma+\beta)} + s_{i}(x_{i+1} - x_{i+1}).
\]

(27)

**Proof**: The proof is straightforward and therefore is omitted.

Without lost of generality, suppose at step \( i = 1 \), there is a \( C^1 \) Lyapunov function such that

\[
V_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) = -\left( n-i+1 \right) \left( x_{i}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{i}^{1(\gamma+\beta)} + s_{i}(x_{i+1} - x_{i+1})
\]

(28)

Using Proposition 1, it is deduced from (28) that

\[
V_i(x_{1}, \ldots, x_i, \hat{t}_i, \hat{\theta}_i) \leq -\left( n-i+1 \right) \left( x_{i}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{i}^{1(\gamma+\beta)} + s_{i}(x_{i+1} - x_{i+1})
\]

(29)

Here, we estimate each term on the right side of (29). Using Lemmas 1–2, we have

\[
\left( x_{i}^{1(\gamma+\beta)} \right)^{i} \leq \frac{1}{4} \left( x_{i}^{1(\gamma+\beta)} \right)^{i} + 4 \left( x_{i}^{1(\gamma+\beta)} \right)^{i} s_{i},
\]

(30)

where \( s_{i} = \left( x_{i}^{1(\gamma+\beta)} \right)^{i} \). To continue the proof, we introduce three additional propositions.

**Proposition 3**: For \( j = 1, \ldots, n \), there are \( C^j \) functions \( \sigma_{j}(x_{1}, \ldots, x_{j}) \geq 0 \) such that

\[
\left| \Delta_{j}(x, t, u) \right| \leq \left( x_{j}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{j}^{1(\gamma+\beta)} \sigma_{j}(\cdot).
\]

(31)

**Proof**: By Lemma 2, for \( l = 2, \ldots, j \),

\[
\left| x_{j} \right| \leq s_{j} + x_{j}^{1(\gamma+\beta)} \leq \left( x_{j}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{j}^{1(\gamma+\beta)} \kappa_{j}(\cdot).
\]

(32)

Using Assumption 1, we have

\[
\left| \Delta_{j}(x, t, u) \right| \leq \left( x_{j}^{1(\gamma+\beta)} \right)^{i} + \ldots + x_{j}^{1(\gamma+\beta)} \kappa_{j}(\cdot) \sigma_{j}(\cdot)
\]

\[
\leq \left( x_{j}^{1(\gamma+\beta)} \right)^{i} + \ldots + \left( x_{j}^{1(\gamma+\beta)} \right)^{i} \sigma_{j}(\cdot),
\]

(33)
where $\sigma_i(.) \geq 0$ is a $C^1$ function.

**Proposition 4:** There is a $C^1$ function $\tau_i(.) \geq 0$ such that

$$\left| \sum_{i=1}^{l} \frac{dW_i}{dx_{ij}} \right| \leq \frac{1}{4} \left[ |\sigma_i|^1 + \left( \sum_{j=1}^{l} |s_j|^1 \right) \right] x_{ij} \tau_i(.) + |s_i|^1 \tau_i(.) .$$

Proof: Using Proposition 3 and Lemma 2, for $l = 2, ..., i-1$, $|\tilde{s}_i| \leq \gamma_j + \sum_{j=1}^{l} |s_j|^1 \tilde{\sigma}_j(.) \leq \left( \sum_{j=1}^{l} |s_j|^1 \right) \tilde{\sigma}_j(.) , (35)

where $\tilde{\sigma}_j(.) \geq 0$ is a $C^1$ function. With the form of $x_{ij}$ in mind, we can prove that

$$d(-x_{ij}) \frac{\partial}{\partial x_{ij}} \leq |s_{i-1}|^1 C_{i-1} .$$

Putting (35) and (36) together and using Lemma 1, we have

$$\left| \sum_{j=1}^{l} \frac{dW_{ij}}{dx_{ij}} \right| \leq |s_{i-1}|^1 C_{i-1} \left( \sum_{j=1}^{l} |s_j|^1 \right) \tilde{\sigma}_j(.) .$$

(37)

where $\tilde{\sigma}_j(.) \geq 0$, $\tau_i(.) \geq 0$ are $C^1$ functions. Now, we are ready to prove Proposition 4. Using (37) and proposition 1,

$$\left| \sum_{i=1}^{l} \frac{dW_i}{dx_{ij}} \right| \leq \sum_{i=1}^{l} \left| x_{ij} - x_{ij} \right| \left| \frac{d(-x_{ij})}{dx_{ij}} \right| \leq \left( 2|s_{i-1}|^1 \sum_{j=1}^{l} |s_j|^1 \right) \sum_{i=1}^{l} \tilde{\sigma}_j(.) .$$

(38)

Using Lemma 1 again, we arrive at (34), which conclude the proof.

**Proposition 5:** There is $C^1$ function $\rho_i(.) \geq 0$ such that

$$\left| \sum_{i=1}^{l} \frac{dW_i}{d\theta_i} \right| \leq \frac{1}{4} \left[ |\sigma_i|^1 + \left( \sum_{j=1}^{l} |s_j|^1 \right) \right] \rho_i(.) + |s_i|^1 \rho_i(.) .$$

Proof: With the form of $\frac{\partial}{\partial \theta_i}$ in mind and using (36), we have

$$\left| \sum_{i=1}^{l} \frac{d(-x_{ij})}{d\theta_i} \right| \leq \frac{1}{4} \left[ |\sigma_i|^1 + \left( \sum_{j=1}^{l} |s_j|^1 \right) \right] \rho_i(.) + |s_i|^1 \rho_i(.) .$$

where $\tilde{\rho}_i(.) \geq 0$ is a $C^1$ function. Using (40), Lemma 1, and Proposition 1,

$$\left| \sum_{i=1}^{l} \frac{dW_i}{d\theta_i} \right| \leq \left( \sum_{i=1}^{l} \left( |s_i|^1 \right) \left( \sum_{j=1}^{l} C_{ij} \right) \right) \left( \sum_{i=1}^{l} \left( |s_i|^1 \right) \right) \tilde{\rho}_i(.) .$$

(41)

Since $(1+Q_i) \geq (1+Q_j)$, we have inequality (39).

Using Proposition 3 and Lemma 1, we get

$$|s_{i-1}|^1 C_{i-1} \left( \sum_{j=1}^{l} |s_j|^1 \right) \tilde{\sigma}_j(.) \leq \left( \sum_{i=1}^{l} \left( - \frac{1}{4} |s_i|^1 \right) \left( \sum_{j=1}^{l} C_{ij} \right) \right) \left( \sum_{i=1}^{l} |s_i|^1 \right) \tilde{\sigma}_j(.) , (39)

(42)

where $\tilde{\sigma}_j(.) \geq 0$, $\tilde{\sigma}_j(.) \geq 0$ are $C^1$ functions. Substituting (30), (34), (39), and (43) into (29), yields

$$\upsilon_i \left( \sum_{i=1}^{l} \beta_i s_i^{(i+1)} \right) + 2 \left( \sum_{i=1}^{l} \beta_i s_i^{(i+1)} \right)^2 \rho_i(.) \leq \left( \sum_{i=1}^{l} \left( - \frac{1}{4} |s_i|^1 \right) \left( \sum_{j=1}^{l} C_{ij} \right) \right) \left( \sum_{i=1}^{l} |s_i|^1 \right) \tilde{\sigma}_j(.) .$$

(44)

Letting $\upsilon_i = \xi_i$ + $\sigma_i(.) + \tau_i(.) + \rho_i(.)$, we get

$$\upsilon_i \left( \sum_{i=1}^{l} \beta_i s_i^{(i+1)} \right) + 2 \left( \sum_{i=1}^{l} \beta_i s_i^{(i+1)} \right)^2 \rho_i(.) \leq \left( \sum_{i=1}^{l} \left( - \frac{1}{4} |s_i|^1 \right) \left( \sum_{j=1}^{l} C_{ij} \right) \right) \left( \sum_{i=1}^{l} |s_i|^1 \right) \tilde{\sigma}_j(.) .$$

(45)

Obviously, the $C^1$ virtual controller

$$x_{i+1} = - \sum_{i=1}^{l} \beta_i s_i^{(i+1)} \left( \sum_{i=1}^{l} \left( - \frac{1}{4} |s_i|^1 \right) \left( \sum_{j=1}^{l} C_{ij} \right) \right) \left( \sum_{i=1}^{l} |s_i|^1 \right) \tilde{\sigma}_j(.) .$$

(46)
with \( \kappa(x_1, \ldots, x, \dot{\theta}) > 0 \) and the updating law
\[
\dot{\theta} = s^{(1+Q)}_{1/2}
\]
result in
\[
\dot{V}_n(x) \leq -\alpha V_n^2 + V_n^2
\]
This completes the proof of the inductive step.

\textbf{Step} \( n \): This is the final step and using the inductive argument above, we have
\[
\dot{V}_n(x) \leq -\alpha V_n^2 + V_n^2
\]
The final controller
\[
\dot{u}_n = -[f(\cdot)+d_{in}(\cdot)] \text{sign}(s_n) - s^{(1+Q)}_{1/2} \gamma (2\beta_x + 2\alpha s^{(1+Q)}_{1/2}) + \sqrt{1+\dot{\theta}_n^2}
\]
with updating law
\[
\dot{\theta}_n = s^{(1+Q)}_{1/2}
\]
where \( \theta_n = 4\beta_x + \alpha s^{(1+Q)}_{1/2} + \gamma \) results in
\[
\dot{V}_n(x) \leq -\alpha V_n^2 - \beta_n V_n^2.
\]

**Theorem 2:** For system (1), if the virtual control laws and the actual control law are designed as (46) and (50), respectively, \( Q_n \) and \( \gamma \) are chosen such that the following inequalities hold
\[
(1+Q_n)\gamma > 1
\]
\[
\frac{(1+Q_n)\gamma - 1}{Q_n} + Q_n > 2
\]
then the state \( x \) will reach zero in finite time \( T \leq \sum_{i=1}^{n} T_i \) and the actual control signal \( u(t) \) is bounded.

**Proof:** By Theorem 1, the sliding mode surfaces \( s_i(x) = 0, \ i = n, \ldots, 1 \), are reached sequentially in finite time \( T \). We obtain \( s_i = x_i = 0, \ i = n, \ldots, 1, \) then \( x_n = \dot{x}_n = -\Delta_n = 0 \). Similarly, \( x_{n-1} = \dot{x}_{n-1} = -\Delta_{n-1} = 0 \).

From (21), since (53) holds and \( Q_n > \ldots > Q_1 \), then the virtual controllers \( x_2, \ldots, x_{n-1} \) are bounded, \( u(t) \) is therefore bounded. From (37), since (54) holds, then \( \frac{d(-x_{i/2})}{dx_i} x_i \) for \( l = 1, \ldots, n-1 \) are bounded, the virtual controllers \( x_2, \ldots, x_{n-1} \) are bounded. Hence, we conclude that the actual control signal \( u(t) \) is bounded.

### III. Simulation Example

To illustrate the effectiveness of the proposed adaptive fast finite time control scheme, consider the following SISO nonlinear uncertain system.

\[
x_1 = x_2 + \epsilon_1 x_1,
\]
\[
x_2 = u + 0.2 \sin(t) + \epsilon_2 (x_1 + x_2) + \epsilon_3 \cos(t),
\]
where \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) are unknown parameters. In the actually simulation, \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) are chosen as 0.1, 0.1, and 0.2, respectively. The initial values of the states are chosen as \( x(0) = 5 \) and \( x(0) = -11 \).

The sliding mode surfaces are designed as \( s_1 = x_1^{19/17}, s_2 = x_2^{19/3} - x_2^{19/3} \). Using (12), (13) in Lemma 3, the virtual control law \( x_{2\ell/2} \) and the update law \( \dot{\theta}_n \) can be designed as
\[
x_{2\ell/2} = -s_1^{(1+17)/(19/21-23)} (2 + 2(3 \times 10^{-6})
\]
\[
+2(7 \times 10^{-7}) s_1^{(1+19/21-23)} + \sqrt{1+\dot{\theta}_n^2},
\]
\[
\dot{\theta}_n = s_1^{(1+19/21-23)},
\]
respectively. Using (50), we have
\[
u = -(0.6 + 0.2 \sin(t)) \text{sign}(s_2) - s_2^{(1+19/3)/(21-23)} (2 \times 10^{-6})
\]
\[
+2(1 \times 10^{-7}) s_2^{(1+19/3)/(21-23)} + \sqrt{1+\dot{\theta}_n^2},
\]
\[
\dot{\theta}_n = s_2^{(1+3)/(21-23)}.
\]

![Fig. 1. The system states](image-url)
Fig. 1 shows the system states $x_1(t)$ and $x_2(t)$, and Fig. 2 shows the phase plot of system states. It is seen that finite time convergence of system states has been achieved using the proposed control scheme.

IV. CONCLUSIONS

In this paper, using the fast non-smooth control Lyapunov function and the method of adding a power integrator merging with adaptive technique, we have successfully developed an adaptive fast finite time control scheme for a class of nonlinear uncertain systems. A simulation example is given in support of the proposed control scheme.

REFERENCES