Signifying quantum benchmarks for qubit teleportation and secure quantum communication using Einstein-Podolsky-Rosen steering inequalities

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The demonstration of quantum teleportation of a photonic qubit from Alice to Bob usually relies on data conditioned on detection at Bob's location. I show that Bohm's Einstein-Podolsky-Rosen (EPR) paradox can be used to verify that the quantum benchmark for qubit teleportation has been reached, without postselection. This is possible for scenarios insensitive to losses at the generation station, and with efficiencies of $\eta_B > 1/3$ for the teleportation process. The benchmark is obtained if it is shown that Bob can "steer" Alice's record of the qubit as stored by Charlie. EPR steering inequalities involving m measurement settings can also be used to confirm quantum teleportation, for efficiencies $\eta_B > 1/m$, if one assumes trusted detectors for Charlie and Alice. Using proofs of monogamy, I show that two-setting EPR steering inequalities can signify secure teleportation of the qubit state.

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I. INTRODUCTION

Quantum teleportation, the process by which a quantum state is transferred from one party to another, has inspired countless investigations and many experiments [1–15]. In a real experiment where imperfections will be present, it becomes necessary to distinguish the process of quantum teleportation from any other process which can be performed classically. The usual procedure is to determine the fidelity $\mathcal{F}$ of the final teleported state relative to the initial state. In a classical process, the final sate is created by a "measure and regenerate" strategy. All such strategies incur extra noise, so that the fidelity cannot exceed a certain value, $\mathcal{F}_c$ [16,17]. The figure of merit for quantum teleportation is a fidelity exceeding $\mathcal{F}_c$.

A very important example of teleportation for the purpose of quantum communication [18,19] is the photonic qubit state teleported over long distances [4,5,14,15]. This quantum teleportation has been realized experimentally using the original protocol of Bennett *et al.* [1]. The criticism has been raised, however, that these experiments may not give truly "loophole-free" demonstrations, since the fidelity is calculated by postselection, i.e., by using only the data observed conditionally on detecting a photon at the teleported location [20]. A fundamental issue is that loss will become more problematic where teleportation distances are large (although the storage of entangled states using quantum memories may overcome this). It is an interesting question, therefore, to ask what levels of overall efficiency can be tolerated in order to claim loophole-free quantum teleportation.

Moreover, the problem of how to demonstrate quantum teleportation is closely linked with how to signify the "security" of the teleported qubit. If Alice teleports a qubit state to Bob, she may want to know that the state is teleported uniquely to him, and not also to another observer, Eve [21–26]. A fidelity $\mathcal{F} > 2/3$ will signify quantum teleportation, which ensures that there is not an infinite number of identical copies of the qubit [24]; the fidelity $\mathcal{F} > 5/6$ will ensure that any "copy" of Bob's qubit held by Eve will have a degraded fidelity (less than 5/6) [21,22]. This knowledge could be used to evaluate the actual security of a string of qubit values that are teleported to Bob by enabling calculation of bounds on Eve's error rate. Security can be measured in terms of the error rate for any possible Eve, *or*, more generally, in terms of the maximum number of Eves that can possess a nondegraded copy of Bob's teleported qubit. However, for such analyses involving lossy systems, the usual approach taken to treat "no detection" events leads to an increase in dimension of the Hilbert space [27–31], so that the original fidelity benchmarks which assume qubit systems are not directly applicable in that case.

In this paper, I present a quite different approach to determining signatures for quantum teleportation. I show how Bohm's Einstein-Podolsky-Rosen (EPR) paradox [32,33] can be used to confirm "loophole-free" without postselection the quantum teleportation and quantum security of a qubit. The result relies on a simple proof of monogamy: Bohm's EPR paradox cannot be shared among more than a finite number of parties. Bohm's EPR paradox is an example of the subclass of nonlocality called "*quantum steering*" [34–36], and the method I propose requires two parties to demonstrate violation of an "EPR steering" inequality [37,38].

I focus on the so-called "entanglement swapping teleportation scenario" [7,39–41]. In that case, Alice's qubit, prior to teleportation to Bob, is entangled with a qubit of Charlie's. This scenario captures the entire teleportation process by including the way Alice's qubit is locally prepared from an EPR-type state. Hence, we are able to address the question raised by Braunstein and Kimble [20] as to whether the zero detection events (if properly accounted for) will detract from the genuine fidelity of the scheme.

We can establish that the quantum benchmark for teleportation has been reached if Bob can demonstrate an EPR paradox, based on his inferred predictions of Charlie's state. This amounts to Bob "steering" Charlie's system (which may be viewed as a record of the qubit teleported by Alice) [41]. We find that quantum teleportation is predicted for arbitrary nonzero efficiencies $\eta_C > 0$ at Charlie's station, and for an overall teleportation efficiency of $\eta_B > 1/3$. These bounds give a sufficient (but not necessary) condition for loophole-free

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quantum teleportation. Furthermore, if we assume trusted detectors at Charlie's station, it would be possible to use the m-setting steering inequalities of Saunders *et al.* [38] and Bennet, Evans *et al.* [29] to confirm quantum teleportation at much lower efficiencies, $\eta_B > 1/m$.

The level of security of the teleported qubit can be determined by the number of measurement settings m associated with the Bohm EPR steering inequality. If Bob demonstrates steering of Charlie's system using an m-setting steering inequality, then there can be a maximum of $m-2$ Eves that can possess identical copies of Bob's qubit. If Bob demonstrates steering of Charlie's system, using a two-setting Bohm EPR inequality, then complete monogamy of the violation of the inequality is guaranteed. I will show that this implies a minimum noise level for the values of Alice's qubits, as inferred by any independent third party (Eve). The violation of the two-setting Bohm EPR inequality is predicted for $\eta_B > 1/2$, provided the assumption of trusted detectors is justified at Charlie's location.

I conclude with a brief discussion, pointing out the "one-sided device-independent" [42,43] nature of the protocol that is proposed in this paper. This means that information is given about the security of the teleported qubit, regardless of the nature of the devices that could be used by the parties, Bob and Eve.

II. DEMONSTRATING BOHM's EPR PARADOX

A. A Bohm EPR paradox criterion

Let us begin with the question of how to confirm Bohm's EPR paradox. This is the case where two systems (A and B) are prepared in the Bell-Bohm state [27,33],

$$|\psi\rangle_s = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\}. \tag{1}$$

The spin outcomes measured at A and B are anticorrelated if the same spin component is measured at each system. Bob at B can make a prediction of *any* Pauli spin component σ_A^θ of Alice's system A by making a measurement on his spin σ_B^θ. According to the EPR premises, usually called "local realism" (LR), this implies a predetermination of *each* of Alice's spin components. In the EPR argument, the predetermined spin components are represented by an "element of reality," which is a hidden variable that defines the spin outcome for Alice's system *precisely*, because Bob's prediction is precise. In the ideal case of Eq. (1), the hidden variable values are 1 or -1. There is inconsistency between the EPR premises and the "completeness of quantum mechanics" because according to LR, all of Alice's spin components are predetermined simultaneously and cannot, therefore, be given by any quantum-mechanical state [32].

In practice, Bob cannot infer Alice's spins with perfect accuracy. We need to know what accuracy will be enough to deduce an EPR paradox. One useful approach is to use quantum uncertainty relations [44]. For three spins, the variances predicted by quantum mechanics for any quantum system A are always constrained to satisfy [45]

$$\left(\Delta\sigma_A^X\right)^2 + \left(\Delta\sigma_A^Y\right)^2 + \left(\Delta\sigma_A^Z\right)^2 \geqslant 2. \tag{2}$$

On recognizing that $\langle(\sigma_{A/B}^\theta)^2\rangle = 1$, we note that this quantum uncertainty relation can also be written as the "circle condition,"

$$\left\langle\sigma_A^X\right\rangle^2 + \left\langle\sigma_A^Y\right\rangle^2 + \left\langle\sigma_A^Z\right\rangle^2 \leqslant 1, \tag{3}$$

used in Refs. [31,46]. By extending the above argument that relates to perfect correlation and ideal states [44,47], we can derive an inequality for a practical test of the Bohm EPR paradox. We demonstrate Bohm's EPR paradox if

$$S_{A|B}^{(3)} = \left(\Delta_{\inf}\sigma_{A|B}^X\right)^2 + \left(\Delta_{\inf}\sigma_{A|B}^Y\right)^2 + \left(\Delta_{\inf}\sigma_{A|B}^Z\right)^2 < 2. \tag{4}$$

Here $\Delta_{\inf}\sigma_{A|B}^X$ is the "inference" uncertainty for Bob's prediction of Alice's spin σ_A^X. When there is a need to specify the second party (in this case B) that is making the inference, we will use the explicit notation $(\Delta_{\inf}\sigma_{A|B}^X)^2$, but otherwise we will write $\Delta_{\inf}\sigma_{A|B}^X$ as $\Delta_{\inf}\sigma_A^X$. The "inference variance" is the average conditional variance

$$\left(\Delta_{\inf}\sigma_{A|B}^X\right)^2 = \sum_{\sigma_B^\varphi=-1,+1} P\left(\sigma_B^\varphi\right)\left\{\Delta\left(\sigma_A^X|\sigma_B^\varphi\right)\right\}^2. \tag{5}$$

This variance gives the uncertainty of the "element of reality" for σ_A^X. Here, $\{\Delta(\sigma_A^X|\sigma_B^\varphi)\}^2$ denotes the variance of the conditional distribution $P(\sigma_A^X|\sigma_B^\varphi)$. The inference variances for the spins σ^Y and σ^Z are defined similarly. We have written (5) as though the best possible prediction for Alice's spin σ_A^X will be given by Bob measuring σ_B^φ, and that this choice will give the smallest $\Delta_{\inf}\sigma_A^X$. The specification of which measurement of Bob's is optimal is irrelevant, however, for the criterion. (For simplicity of notation, we assume it is understood from the context whether we are referring to the spin operator measurements $\hat{\sigma}_{A/B}^\theta$ or the outcomes of those measurements, and we omit the "hats" in the first case.)

When the criterion (4) is achieved, the inferred uncertainties "violate" the uncertainty principle (2) if they represent simultaneous descriptions of spin components. For this reason, the inequality (4) will demonstrate the incompleteness of quantum mechanics based on the assumption of LR [44,47]. The inequality is thus a sufficient condition for Bohm's EPR paradox.

Bohm's EPR paradox inequality is closely related to the steering inequality used by Wittmann *et al.* to demonstrate loophole-free steering. The close relationship between the EPR paradox and quantum steering was pointed out in Refs. [35–37]. Since $(\Delta_{\inf}\sigma_{A|B}^\theta)^2 = 1 - \langle\sigma_A^\theta|\sigma_B^\varphi\rangle^2$, where $\langle\sigma_A^\theta|\sigma_B^\varphi\rangle$ is the mean of $P(\sigma_A^X|\sigma_B^\varphi)$, substitution into (4) yields the equivalent inequality

$$S = T_X + T_Y + T_Z > 1, \tag{6}$$

where $T_X = \sum_{\sigma_B^\varphi} P(\sigma_B^\varphi)\langle\sigma_A^X|\sigma_B^\varphi\rangle^2$ (and similarly for T_Y and T_Z). This is precisely the steering inequality used by Wittmann *et al.* [31].

B. Bohm's EPR paradox without fair sampling assumptions

The inequality (4) does not take into account detection losses, where one or both of the particles is not detected. The usual procedure is to introduce a fair sampling assumption, where all "no detection" events are ignored. In some recent experiments that detect (without fair-sampling loopholes) the sort of nonlocality called "quantum steering," the assumption is made that the detectors for Alice's particle can be "trusted" [29–31]. This means that the fair sampling assumption is made asymmetrically for Alice's system but not for Bob's.

The Bohm EPR condition (4) can be modified so that it will apply without fair sampling assumptions for either party. This provides a way to demonstrate the "loophole-free" Bohm EPR paradox. The original condition (4) is derived from the quantum uncertainty relation (2) which is valid only when the outcomes for the measurements are dichotomic (± 1). This uncertainty relation can be modified to allow for "no detection" events, which are labeled by an outcome of 0. This approach of expanding the Hilbert space has been commonly used to treat the effect of loss on nonlocality [27–31].

It is convenient to introduce the Schwinger formalism for spins. This enables a direct analogy with the photonic realization of spin measurements, whereby the Stern-Gerlach apparatus is replaced by polarizing beam splitters. Two orthogonal polarization field modes are defined at each of two sites A and B, and are identified by boson operators $a_\pm$ and $b_\pm$, respectively. In the ideal case, the spin states at A are $|\uparrow\rangle_A = |1\rangle_{a+}|0\rangle_{a-}$ and $|\downarrow\rangle_A = |0\rangle_{a+}|1\rangle_{a-}$, which describe a photon in one of the polarization modes. More generally, the measurable Schwinger spin observables at A are

$$\begin{aligned} S_A^Z &= a_+^\dagger a_+ - a_-^\dagger a_-, \quad S_A^X = a_+^\dagger a_- + a_+ a_-^\dagger, \\ S_A^Y &= (a_+^\dagger a_- - a_+ a_-^\dagger)/i, \quad S_A^2 = n_A(n_A+2), \\ n_A &= a_+^\dagger a_+ + a_-^\dagger a_-, \end{aligned} \tag{7}$$

where $S_A^2 = (S_A^X)^2 + (S_A^Y)^2 + (S_A^Z)^2$, and n_A is the total number operator. Similar operators and states are defined for the system B.

Having established the formalism, we now introduce an uncertainty relation

$$\left(\Delta S_A^X\right)^2 + \left(\Delta S_A^Y\right)^2 + \left(\Delta S_A^Z\right)^2 \geqslant \left\langle n_A^2\right\rangle - \langle n_A\rangle^2 + 2\langle n_A\rangle \tag{8}$$

which will hold for any quantum state, and which follows from $\langle S^2\rangle = \langle n(n+2)\rangle$ and that $\langle S_X\rangle^2 + \langle S_Y\rangle^2 + \langle S_Z\rangle^2 \leqslant \langle n\rangle^2$ [46,48]. This uncertainty relation can be used to derive an inequality for the Bohm EPR paradox in nonideal scenarios.

Specifically, by applying the EPR argument with the quantum uncertainty relation (8), we see that we will verify an EPR paradox if

$$\begin{aligned} &\left(\Delta_{\inf} S_A^X\right)^2 + \left(\Delta_{\inf} S_A^Y\right)^2 + \left(\Delta_{\inf} S_A^Z\right)^2 \\ &\quad < \left\langle n_A^2\right\rangle - \langle n_A\rangle^2 + 2\langle n_A\rangle. \end{aligned} \tag{9}$$

Here, we define

$$\left(\Delta_{\inf} S_{A|B}^X\right)^2 = \sum_{s_B^\varphi = -1,0,+1} P\left(s_B^\varphi\right)\left\{\Delta\left(S_A^X \middle| S_B^\varphi\right)\right\}^2 \tag{10}$$

in accordance with definition (5). This inequality is the generalization of the EPR Bohm paradox condition (4) that accounts for detection inefficiencies, at both sites, and is the main result of this paper.

The inequality is also a "steering" inequality, and can be derived directly from the local hidden state formalism established in Refs. [35,36]. This proof is presented in the Appendix. We will use the term "EPR steering" to refer to such inequalities that test both the EPR paradox and the nonlocality of quantum steering [37].

With only one photon incident at each site, and the possibility of "no detection," the possible outcomes for a given "spin" $S_{A/B}^{Z/Y/X}$ are $+1$, -1, and 0. Denoting the probabilities for each of these outcomes at site j ($j \equiv A,B$) by P_{+j}, P_{-j}, and P_{0j}, respectively, we note that

$$\langle n_j\rangle = P_{+j} + P_{-j} = \eta_j,$$

where η_j is the efficiency at the site j. We use the notation η_A for the efficiency at site A, and η_B for the efficiency at site B.

We can also modify the steering inequality used by Wittmann *et al.* [31], so that it accounts for the inefficiencies at the "trusted site" of Alice. Since the outcomes for each $S_{A/B}^\theta$ are ± 1 or 0 (here $\theta \equiv X,Y,Z$), it is easy to verify that $\langle (S_A^\theta)^2\rangle = \eta_A$, and hence that $(\Delta_{\inf} S_{A|B}^X)^2 = \sum_{s_B^X} P(s_X^B)\{\eta_A - \langle S_A^X|S_B^X\rangle^2\} = \eta_A - T_X$, where here $\langle S_A^X|S_B^X\rangle$ denotes the mean value of S_A^X given the result S_B^X. Thus, the Bohm EPR condition (9) can be written

$$S = T_X + T_Y + T_Z > \eta_A^2, \tag{11}$$

which is the extension of the steering inequality (6) used by Wittmann *et al.* [31]. If this inequality is satisfied, then one can confirm a steering of system A by measurements performed by Bob (at system B) *without* the assumption of trusted detectors at Alice's location.

C. Quantum prediction

We now ask, for what quantum states and with what degree of loss can the Bohm EPR paradox criterion be satisfied? Let us assume the system is in a Werner mixed state:

$$\hat{\rho} = (1 - p_s)\tfrac{1}{4}I + p_s|\psi\rangle_{S\,S}\langle\psi|, \tag{12}$$

where p_s gives the relative contribution of the Bell state $|\psi_S\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\}$ and I is a rotationally symmetric, uncorrelated state proportional to the identity matrix at each site. The calculations are given in the Supplemental Material [49]. We find that for a system in the Werner state and with detection efficiencies η_A and η_B at each site, the quantum prediction is

$$\left(\Delta_{\inf} S_{A|B}^\theta\right)^2 = \eta_A\left\{1 - \eta_A\eta_B p_s^2\right\}, \tag{13}$$

where $\theta \equiv X,Y,Z$. The Bohm EPR condition (9) is satisfied when

$$\eta_B > 1/\left(3p_s^2\right), \tag{14}$$

i.e., for $\eta_B > 1/3$, where $p_s = 1$ (provided $\eta_A > 0$). This efficiency for η_B (Bob's detection) has been achieved in the experiments of Wittmann *et al.* [31]. We note that the criterion is satisfied *independently* of the value of the efficiency for Alice's detection, so long as it is nonzero.

The EPR steering inequality (9) is very useful for loophole-free tests since it applies regardless of the photon numbers *actually incident* on the detectors. This means it can fully account for all spurious events. This is important when the photon pairs are generated via parametric downconversion, since then there is always a possibility of two photon pairs being generated. As these events usually occur with a very small probability, however, the quantum prediction given here is valid for most scenarios.

III. SIGNATURE FOR QUBIT QUANTUM TELEPORTATION WITHOUT FAIR SAMPLING

We now address the question of how to apply the Bohm EPR criterion to demonstrate quantum teleportation. We begin with a simple proof of monogamy. If a party B can demonstrate a steering of the party A by satisfying the Bohm EPR paradox inequality (4) or its generalization (9), then there cannot be an infinite number of other parties that can also do this.

A. Monogamy relations for the EPR steering inequalities

We define a "steering parameter" that is based on the Bohm EPR criterion (9):

$$S_{A|B}^{(3)} = \left\{\left(\Delta_{\inf} S_A^X\right)^2 + \left(\Delta_{\inf} S_A^Y\right)^2 + \left(\Delta_{\inf} S_A^Z\right)^2\right\}/J, \qquad (15)$$

where $J = \langle n_A^2\rangle - \langle n_A\rangle^2 + 2\langle n_A\rangle$. Then we see that according to (9), EPR steering of system A by B is obtained when $S_{A|B}^{(3)} < 1$. We note that this inequality involves three observables, and is hence a "three-setting inequality."

We now prove that a monogamy steering relation holds for the steering parameter. For any four quantum systems A–D, it is always true that

$$S_{A|B}^{(3)} + S_{A|C}^{(3)} + S_{A|D}^{(3)} \geqslant 3. \qquad (16)$$

This result, and a collection of other monogamy results for the EPR paradox and quantum steering, have been presented and proved in previous papers [50,51]. The proof is briefly summarized here for the sake of completeness.

Proof. The observer at B (Bob) can make the measurement that gives him the value of Alice's observable S_A^X with uncertainty $\Delta_{\inf} S_{A|B}^X$. The observer at C (Charlie) can make the measurement that gives the result for Alice's S_A^Y with uncertainty $\Delta_{\inf} S_{A|C}^Y$, *and* the observer at D can make the measurement that gives the result for Alice's S_A^Z with uncertainty $\Delta_{\inf} S_{A|D}^Z$. Since the three observers can measure simultaneously, the uncertainty relation (8) constrains the variances to be $(\Delta_{\inf} S_{A|B}^X)^2 + (\Delta_{\inf} S_{A|C}^Y)^2 + (\Delta_{\inf} S_{A|D}^Z)^2 \geqslant J$. Similarly, $(\Delta_{\inf} S_{A|D}^X)^2 + (\Delta_{\inf} S_{A|B}^Y)^2 + (\Delta_{\inf} S_{A|C}^Z)^2 \geqslant J$ and also $(\Delta_{\inf} S_{A|C}^X)^2 + (\Delta_{\inf} S_{A|D}^Y)^2 + (\Delta_{\inf} S_{A|B}^Z)^2 \geqslant J$. We then see that the monogamy relation (16) follows, upon adding the three inequalities. ■

The monogamy result (16) tells us that, within the constraints of quantum theory, it is impossible for more than two parties to (independently) demonstrate the steering of system A by the procedure of violating the three-setting Bohm EPR paradox inequality (9).

B. Quantum teleportation of a qubit

There is a close relationship between monogamy and quantum no-cloning. We now turn to the situation in which Alice teleports a quantum state to Bob via an entanglement swapping protocol. The monogamy of the three-observable steering inequality (9), as given by (16), will restrict the number of equivalent copies of Alice's state that can be teleported to different parties. Such a restriction cannot be achieved by any classical "measure and regenerate" strategy, since such a strategy would allow an infinite number of equivalent copies to be regenerated.

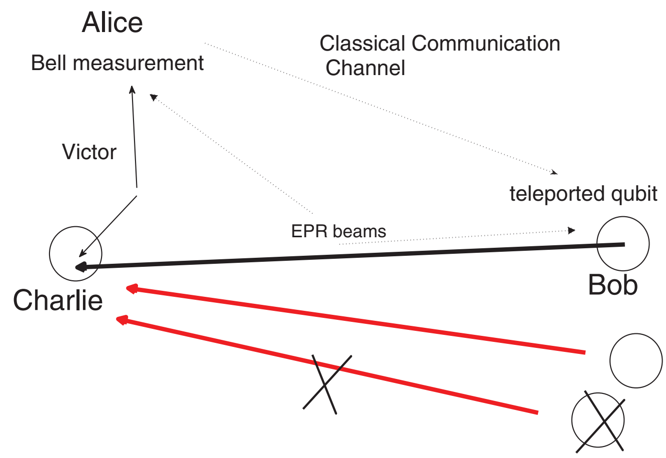

FIG. 1. (Color online) Schematic of verification of quantum teleportation using the monogamy inequalities. If Bob can verify steering of Charlie's qubit system, using a two-observable steering inequality, then no other party (Eve) can also do this. This excludes the possibility of a clone of the teleported state, and it gives confirmation of secure quantum teleportation. If Bob can steer Charlie's qubit system using an m-setting steering inequality, then there can be no more than $m-2$ clones (in the diagram, $m=3$, and the red lines indicate the possible "Eves"): the possibility of a classical "measure and regenerate" strategy is negated, and quantum teleportation confirmed.

Let us consider the setup of Fig. 1, where a Bohm EPR two-qubit state is prepared at the site of Alice, Charlie, and Victor. One qubit is with Charlie. The second EPR qubit is with Victor, and is then teleported to Bob by Alice's sending station. After teleportation, the entanglement is "swapped" and Bob and Charlie share an entangled EPR state [7,39–41].

In the standard protocol, two EPR beams are prepared in the Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\}$. One of these beams is sent to Alice, the other to Bob. Victor and Charlie prepare an entangled EPR qubit,

$$|\phi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_V^\theta|\downarrow\rangle_C^\theta - |\downarrow\rangle_V^\theta|\uparrow\rangle_C^\theta\}, \qquad (17)$$

and Victor will teleport his qubit to Bob. Victor's qubit is input to a teleportation device, while the correlated qubit remains with Charlie. Alice performs a Bell measurement on the direct product state which can be written as a linear combination of the four Bell states [1]. If Alice measures the system to be in a Bell state $|\Psi_-\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_V^\theta|\downarrow\rangle_A^\theta - |\downarrow\rangle_V^\theta|\uparrow\rangle_A^\theta\}|\Psi_-\rangle$, then her result is sent classically to Bob, who recovers Victor's qubit by performing the identity operation on his state. The procedure swaps the entanglement between Charlie and Victor to one between Charlie and Bob, and the final state is the entangled qubit,

$$\frac{1}{\sqrt{2}}\{-|\uparrow\rangle_B|\downarrow\rangle_C + |\downarrow\rangle_B|\uparrow\rangle_C\}. \qquad (18)$$

This description summarizes the entire teleportation process, even when viewed as teleportation of a single qubit, since in practice the correlated Bell-Bohm EPR state of Victor and Charlie is used to herald the qubit that is input to Alice's sending station [4].

If Charlie has detected his qubit (before teleportation so both he and Alice know the qubit value and angle θ), then we may view the process as teleportation of a qubit. Victor's input to the sending station is the anticorrelated eigenstate of definite spin along the direction θ. If Alice performs a Bell measurement with result indicating $|\Psi_-\rangle$ and transmits this result indicating to Bob, then he will know that his state is the same state as Victor's input. Alternatively, if Charlie does not make his measurement, then Bob's state is entangled with Charlie's state. In that case, if Charlie measures his spin along the direction θ, then he and Bob will know that Bob's state is the eigenstate with the anticorrelated spin along the same direction. Delayed-choice entanglement swapping has been studied in recent experiments [41,52].

C. Signature of quantum teleportation

To claim quantum teleportation, there must be a signature to indicate that Bob's final teleported state cannot be generated using any classical strategy. Usually this is done by demonstrating that the fidelity with Alice's input state is higher than can be achieved based on any classical "measure and regenerate" protocol. It has been proved that according to quantum mechanics a high fidelity, $\mathcal{F} > 2/3$, can only be achieved for a finite number of copies of a qubit state. This feature is a consequence of the quantum no-cloning theorem [16,21,22,53]. Since for any classical teleportation strategy an infinite number of identical copies are possible, the criterion $\mathcal{F} > 2/3$ will demonstrate quantum teleportation for qubit systems. In fact, this fidelity criterion is a necessary and sufficient condition, and has been used to determine the optimal teleportation with mixed-state qubits [54].

Here, I consider a different approach, based on an EPR steering inequality. Bob measures the value of the teleported qubit along a direction θ. Charlie communicates classically the value of θ that he or Victor used to define the qubit, so that Bob knows what measurement to make. Alice and Charlie know when she sent the information about the qubit. The experiment involves the quantum state conditional on her making the Bell measurement and sending the classical signal to Bob.

For a sequence of states each with the value of θ, Bob reports to Charlie his values, Charlie performs the same measurement of his spin, and the conditional variance $(\Delta_{\inf}\sigma^{\theta}_{C|B})^2$ is measured. If this is done for three orthogonal selections of θ, the steering parameter $S^{(3)}_{C|B}$ can be determined. The observation of steering is given when $S^{(3)}_{C|B} < 1$. Suppose Bob and Charlie verify that Bob has satisfied the EPR steering criterion

$$S^{(3)}_{C|B} < 1. \tag{19}$$

Use of the monogamy relation $S^{(3)}_{C|B} + S^{(3)}_{C|D} + S^{(3)}_{C|E} \geqslant 3$ [Eq. (16)] tells us that there can be no more than one other party that can also show a steering of Charlie's system by way of this criterion, i.e., for independent parties C, D, and E, if $S^{(3)}_{C|D} < 1$, then we know that $S^{(3)}_{C|E} \geqslant 1$. This excludes the possibility of more than one clone produced by the teleportation process, since a second clone at E would be able to establish the same value of $S^{(2)}_{C|E} < 1$, which contradicts the monogamy result (16). If EPR steering is verified by $S^{(3)}_{C|B} < 1$, then quantum teleportation of Bob's state is verified, since any classical "measure and regenerate" strategy to generate that state would enable an infinite number of identical states to be produced on teleportation.

The EPR steering inequality (19) is thus a *sufficient condition* to demonstrate quantum teleportation. We note that the ideal transmission of every qubit will lead to $S^{(3)}_{C|B} \to 0$, so that the quantity defined by taking the maximum of 0 or $1 - S^{(3)}_{C|B}$ gives a type of "figure of merit" for the teleportation process. This is not a true figure of merit for quantum teleportation itself, however, as the inequality is a sufficient but not necessary condition for quantum teleportation (as we will see in Sec. V). Other EPR steering inequalities have been derived, for example, based on entropic uncertainty relations [55], which could give a more effective test of the steering.

The important point is that the three-observable steering inequality $S^{(3)}_{C|B} < 1$ is achievable at quite low inefficiencies for the qubit Bell state [Eq. (18)] shared between Charlie and Bob. Let η_C be Charlie's efficiency and η_B be Bob's efficiency. The predictions given in Sec. II C are $(\Delta_{\inf}S^{\theta}_{C|B})^2 = \eta_C\{1 - \eta_B\eta_C\}$ $(\theta = X,Y,Z)$. The Bohm EPR condition is satisfied when $\eta_B > 1/3$ (provided $\eta_C > 0$). The loophole-free verification is insensitive to the losses η_C of Charlie's detectors. The efficiency $\eta_B > 1/3$ is difficult to achieve with current technology because it represents the entire efficiency of the teleportation process, from Charlie to Bob and including Bob's detection inefficiency. This is because of the significant losses that take place at Alice's sending station.

However, we can define and consider the quantum state $\rho_{CB|A}$ of Charlie and Bob, *conditional* on Alice making a successful Bell measurement and sending the classical information. In this scenario, it is envisaged that Charlie has not made his measurement, but the information about Alice's qubit is stored in Charlie's spin-1/2 system. This is the case of entanglement swapping teleportation. Since $\rho_{BC|A}$ is a quantum state, the monogamy relation is predicted to hold for Bob and Charlie's measurements (on this conditional state), and therefore the inequality will remain a signature of quantum teleportation. In that case, we can argue that we have gained confirmation of quantum teleportation regardless of inefficiencies at Alice's sending station. The requirement of $\eta_B > 1/3$ is determined by the efficiency of Bob's detection and the losses on Bob's EPR channel only. This level of efficiency has been realized in the loophole-free steering experiments of Wittmann *et al.* [31] and would appear to be quite feasible.

In many situations, the EPR channels of Alice and Bob are propagated from a common source. To achieve true quantum teleportation for that case, since the sensitivity is with respect to Bob's efficiency η_B, the EPR source is best placed close to Bob's station in order to minimize losses on Bob's channel. It also becomes essential that he has the best detectors. These sorts of issues are discussed in Ref. [43] from the perspective of quantum key distribution (QKD).

IV. SECURE QUBIT TELEPORTATION USING TWO-SETTING EPR STEERING INEQUALITIES

Let us consider the experiment in which Bob and Charlie are able to demonstrate that the EPR steering inequality (19) is satisfied. Then they can confirm the quantum benchmark for teleportation. The monogamy relation (16) gives us a *stronger* result: there can be *no more* than one party E (other than Bob) also able to satisfy the EPR inequality, $S_{C|E}^{(3)} < 1$. On examining the proof of the monogamy relation, we see that this follows because the EPR inequality involves three observables, and hence there are three measurement settings at each location. The result directly implies a level of security of the qubit values shared by Charlie and Bob, because the EPR inequality can only be satisfied if the variances in the inferences of Charlie's qubit values are small enough. The inference variances for the other parties must be large, and are quantifiable using the monogamy relation (16).

We note that we can improve the level of security if Bob and Charlie use *two-setting* EPR steering inequalities. In that case, there can be *no* party other than Bob that can demonstrate the EPR steering inequality. Two-setting inequalities for Pauli spins have been derived in Refs. [37,38,46]. Here, we consider a two-setting EPR inequality expressed in terms of the conditional variances. We find that EPR steering of Charlie's system C by Bob's measurements at B is observed if

$$\left(\Delta_{\inf}\sigma_{C|B}^{X}\right)^2 + \left(\Delta_{\inf}\sigma_{C|B}^{Y}\right)^2 < 1. \tag{20}$$

The proof is presented in Ref. [37] and is outlined in the Appendix. This inequality is also a condition for Bohm's EPR paradox. Introducing the steering parameter $S_{C|B}^{(2)} = (\Delta_{\inf}\sigma_{C|B}^{X})^2 + (\Delta_{\inf}\sigma_{C|B}^{Y})^2$, we can write the monogamy relation

$$S_{C|B}^{(2)} + S_{C|E}^{(2)} \geqslant 2 \tag{21}$$

that follows on extending the results and definitions of (16). The relation has been derived in Ref. [51] and will *always* hold. Thus, if Bob can demonstrate $S_{C|B}^{(2)} < 1$, then this ensures that for any other party E (Eve), it is the case that $S_{C|E}^{(2)} \geqslant 1$, which implies a minimum noise level on Eve's inference of Charlie's qubit values. Thus, the two-setting inequality $S_{C|B}^{(2)} < 1$ confirms what we will call interchangeably "faithful teleportation" or "secure teleportation."

The two-setting EPR inequality (20) is derived based on the uncertainty relation $(\Delta\sigma_C^X)^2 + (\Delta\sigma_C^Y)^2 \geqslant 1$ for Pauli spins, which holds for any quantum state. The inequality, therefore, also holds for the Schwinger spins defined in (7) but provided the outcomes for Charlie's spins (at C) are ± 1. Assuming perfect detectors at Charlie's station, and assuming the system is prepared in a maximally correlated Bell state, it is easy to show from the results of Sec. II C and the Supplemental Material [49] that the inequality (20) can be satisfied for any $\eta_B > 1/2$.

In short, the inequality $S_{C|B}^{(2)} < 1$ can be used to confirm secure teleportation provided one assumes *trusted* detectors at Charlie's measurement station, so that the fair sampling assumption is justified at this location. In that case, security of the teleported state can be confirmed for up to 50% losses in the teleportation process (i.e., for Bob's channel and detectors).

V. DEMONSTRATION OF QUANTUM TELEPORTATION AT ARBITRARY EFFICIENCIES

A set of EPR inequalities has been derived that involve m settings [29,38]. These inequalities can be expressed in a form similar to the steering inequalities (9) and (20), which we write as $S_{C|B}^{(3)} < 1$ and $S_{C|B}^{(2)} < 1$. If $S_{C|B}^{(m)} < 1$, then it is confirmed that Bob can steer Charlie's system using an m-setting inequality. The exact form of $S_{C|B}^{(m)}$ is given by the results in Refs. [29,38].

It has been shown that the m-setting inequalities also satisfy a monogamy relation [51]. If Bob and Charlie can demonstrate an m-setting steering inequality to confirm that Bob can steer Charlie's system, then there can be no more than $m-2$ parties (other than Bob) that can also demonstrate the m-setting inequality. The realization of the inequality $S_{C|B}^{(m)} < 1$ therefore confirms quantum teleportation. This is because a classical protocol would enable generation of an infinite number of identical teleported states, which in turn enables an infinite number of parties to demonstrate the inequality, in contradiction with the monogamy result. The value $m-2$, which gives the maximum number of parties (Eve) that can possess an identical copy of Bob's state, is an indicator of the quality of the teleportation.

It is possible to evaluate for what efficiencies the m-setting inequalities can be satisfied, assuming Bob and Charlie share a Bell state. With the assumption that Charlie has "trusted detectors" (i.e. maximum efficiency $\eta_C = 1$), it has been shown by Evans *et al.* [29] that the inequality $S_{C|B}^{(m)} < 1$ can be satisfied for optimal measurement choices provided $\eta_B > 1/m$. This is an important result that indicates quantum teleportation can be demonstrated *for arbitrary losses* at Bob's receiving station, provided we can make the assumption of fair sampling at the generation (Charlie) stage.

VI. BRAUNSTEIN-KIMBLE CRITICISM

Braunstein and Kimble have commented that the quality of qubit quantum teleportation is limited by the "no detection" outcomes at Bob's location [20]. They also point out that the low efficiency for generation of the EPR pair when using parametric amplification would lead to a high incidence of "no detection" outcomes, even with ideal detectors. As mentioned by them, these problems can be overcome, e.g., by heralding the EPR pair [15,39]. In terms of establishing a deterministic teleportation, high efficiency of the teleportation process is essential [6].

However, it remains interesting to ask whether the claim of quantum teleportation is compromised in the presence of zero detections. We have established that quantum teleportation can in principle be demonstrated for quite low efficiencies. Lastly, we address the second question for the scenario considered in this paper. We examine the effect of the vacuum state that arises in the parametric process that generates the photonic EPR pair. We consider that the actual EPR resource is the quantum state of the four-mode parametric amplifier:

$$|\psi\rangle = c_0|0\rangle_{a+}|0\rangle_{a-}|0\rangle_{b+}|0\rangle_{b-} + c_1\frac{1}{\sqrt{2}}\{|1\rangle_{a+}|0\rangle_{a-}|1\rangle_{b+}|0\rangle_{b-} + |0\rangle_{a+}|1\rangle_{a-}|0\rangle_{b+}|1\rangle_{b-}\}. \tag{22}$$

Here, the contributions of terms involving modes with two or more photons have been ignored, and therefore $|c_0|^2 + |c_1|^2 = 1$. For simplicity, we consider the case in which there are no losses. Since a register for Alice's Bell measurement requires a coincidence at her detectors (for the detection of the $|\Psi_-\rangle$ Bell state), we note that Alice's classical signal for teleportation go-ahead will always be correlated with a detection of a photon at Bob's detector in that case. This means that the vacuum state has no effect on the calculations presented for that particular scenario.

VII. CONCLUSION

The objective of this paper is to propose alternative ways to signify the quantum teleportation of a qubit that can be applied without postselection. Two sorts of inequalities have been presented. The first is a single inequality that allows confirmation of quantum teleportation without fair sampling assumptions at either station: where the qubit is generated (Charlie), or where it is detected after teleportation (Bob). This inequality involves three measurement settings and can give a demonstration of Bohm's EPR paradox, and quantum teleportation, for efficiencies $\eta_C > 0$ at Charlie's station and $\eta_B > 1/3$ at Bob's station. In this case, the quantum state that is considered as being teleported is that conditioned on Alice's successful performance of the Bell measurement, and the teleportation protocol is one of entanglement swapping. The proof of quantum teleportation is based on a proof of monogamy of the EPR paradox.

The second sort of inequality can be used where a fair sampling assumption is made at Charlie's station, so that the outcomes of his measurements are confined to the qubit Hilbert space. Such an assumption has been called that of "trusted detectors." In that scenario, steering of Charlie's system by Bob (and hence quantum teleportation) can potentially be demonstrated for arbitrary efficiency at Bob's detectors. This conclusion is based on the results of Bennet, Evans *et al.* [29], which report steering inequalities to be violated for efficiencies $\eta_B > 1/m$, where m is the number of measurement settings.

An important example of this second sort of inequality is a Bohm's EPR paradox inequality for two settings ($m = 2$). This inequality requires efficiencies of $\eta_B > 1/2$ (for correlations based on the maximally entangled Bell state). The useful feature of the two-setting inequality is that a high level of security of the teleported qubit state can be deduced. There can be no other independent party (Eve) also able to demonstrate the inequality (apart from Bob), which implies a minimum noise level on Eve's inferences of Charlie's qubit value.

We have seen that the EPR and steering paradoxes are useful to demonstrate quantum teleportation in the presence of loss. Are there other advantages? A possible response relates to the nature of the derivation of the EPR steering inequalities. It is assumed that Charlie's measurements are made on a quantum spin system, and are therefore constrained by quantum mechanics. However, the EPR steering inequalities are derived with *no* similar assumption about what or how measurements are made by Bob (or Eve) [42]. In this way, we see that the conditions for quantum teleportation (and for the level of security of the teleported state) have the advantages of being "one-sided device-independent" [43].

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APPENDIX

We give a derivation of the EPR paradox and steering inequalities. This type of proof has been presented in Ref. [37]. We begin with the definition of the local hidden state (LHS) model [35,36]. To prove steering, we need to falsify a description of the statistics based on a LHS model, where the averages are given as

$$\langle X_B X_A\rangle = \int_R P_R \langle X_B\rangle_R \langle X_A\rangle_{R,\rho}. \tag{A1}$$

Here $\int_R P_R = 1$, and the ρ subscript indicates that the averages are consistent with those of a quantum density matrix. No such constraint is made for the moments $\langle X_i\rangle_R$, written without the subscript ρ. This model is one in which the system is a probabilistic mixture of states symbolized by R, with probabilities P_R. The states symbolized by R (without the subscript ρ) may be identified as the local hidden variable states assumed in Bell's local hidden variable models. The summation over all possible states R can be denoted either by an integral or by a discrete summation, similar to the situation for Bell's local hidden variable models [27].

The average conditional uncertainty is

$$\left(\Delta_{\inf}\sigma_A^X\right)^2 = \sum_{x_j^B} P\left(x_j^B\right)\left\{\Delta\left(\sigma_A^X\middle|x_j^B\right)\right\}^2, \tag{A2}$$

where we denote the possible results of the specified measurement at B by $\{x_j^B\}$. Using the definitions, and assuming the mixtures as implied by the LHS model, we see step by step that

$$\begin{aligned}
&\sum_{x_j^B} P\left(x_j^B\right)\left\{\Delta\left(\sigma_A^X\middle|x_j^B\right)\right\}^2\\
&= \sum_{x_j^B} P\left(x_j^B\right)\sum_{\sigma_x^A} P\left(\sigma_A^X\middle|x_j^B\right)\left\{\sigma_A^X - \left\langle\sigma_A^X\middle|x_j^B\right\rangle\right\}^2\\
&= \sum_{x_j^B,\sigma_A^X} P\left(x_j^B,\sigma_A^X\right)\left\{\sigma_A^X - \left\langle\sigma_A^X\middle|x_j^B\right\rangle\right\}^2\\
&= \sum_R P_R \sum_{x_j^B,\sigma_A^X} P_R\left(x_j^B,\sigma_A^X\right)\left\{\sigma_A^X - \left\langle\sigma_A^X\middle|x_j^B\right\rangle\right\}^2\\
&\geqslant \sum_R P_R \sum_{x_j^B} P_R\left(x_j^B\right)\sum_{\sigma_A^X} P_R\left(\sigma_A^X\middle|x_j^B\right)\left\{\sigma_A^X - \left\langle\sigma_A^X\middle|x_j^B\right\rangle_R\right\}^2\\
&= \sum_R P_R \sum_{x_j^B} P_R\left(x_j^B\right)\left\{\Delta_R\left(\sigma_A^X\middle|x_j^B\right)\right\}^2\\
&= \sum_R P_R\left\{\Delta_{\mathrm{inf,R}}\sigma_A^X\right\}^2.
\end{aligned}$$

The fourth line follows using that for a probabilistic mixture $P(x_j^B,\sigma_X^A) = \sum_R P_R P_R(x_j^B,\sigma_X^A)$. The fifth line follows from the fact that $\langle(x-\delta)^2\rangle \geqslant \langle[x-\langle x\rangle]^2\rangle$, where δ is any number. Here, the subscripts R imply that the probabilities, averages, and variances are with respect to the state R. Now, if we assume the separability between the bipartition A-B for each state R, in accordance with the LHS model, then

$$P_R\left(x_j^B,\sigma_A^X\right) = P_R\left(x_j^B\right)P_R\left(\sigma_A^X\right). \tag{A3}$$

This implies $\langle\sigma_A^X|x_j^B\rangle_R = \langle\sigma_A^X\rangle$ and $\{\Delta_R(\sigma_A^X|x_j^B)\}^2 = \{\Delta_R(\sigma_A^X)\}^2$. Then we find, on using $\sum_{x_j^B} P_R(x_j^B)=1$, that we can write $\{\Delta_{\inf,\mathrm{R}}\sigma_A^X\}^2 = \{\Delta_R(\sigma_A^X)\}^2$. Thus,

$$\begin{aligned}&\left(\Delta_{\inf}\sigma_A^X\right)^2 + \left(\Delta_{\inf}\sigma_A^Y\right)^2\\ &\quad\geqslant \sum_R P_R\left[\left\{\Delta_R\left(\sigma_A^X\right)\right\}^2 + \left\{\Delta_R\left(\sigma_A^Y\right)\right\}^2\right]\end{aligned}$$

and

$$\begin{aligned}&\left(\Delta_{\inf}\sigma_A^X\right)^2 + \left(\Delta_{\inf}\sigma_A^Y\right)^2 + \left(\Delta_{\inf}\sigma_A^Z\right)^2\\ &\quad\geqslant \sum_R P_R\left[\left\{\Delta_R\left(\sigma_A^X\right)\right\}^2 + \left\{\Delta_R\left(\sigma_A^Y\right)\right\}^2 + \left\{\Delta_R\left(\sigma_A^Z\right)\right\}^2\right].\end{aligned}$$

Because in the LHS model (A1) we assume the states at A are local quantum states, we can use quantum uncertainty relations to derive a final steering inequality: e.g., $\{\Delta_R(\sigma_A^X)\}^2 + \{\Delta_R(\sigma_A^Y)\}^2 \geqslant 1$ for any quantum state, and hence the LHS model implies

$$\left(\Delta_{\inf}\sigma_A^X\right)^2 + \left(\Delta_{\inf}\sigma_A^Y\right)^2 \geqslant 1. \tag{A4}$$

Also, the uncertainty relation (8) will hold for any quantum state. Thus the LHS model implies

$$\begin{aligned}&\left(\Delta_{\inf}S_A^X\right)^2 + \left(\Delta_{\inf}S_A^Y\right)^2 + \left(\Delta_{\inf}S_A^Z\right)^2\\ &\quad\geqslant \left\langle n_A^2\right\rangle - \langle n_A\rangle^2 + 2\langle n_A\rangle.\end{aligned} \tag{A5}$$

Violation of either of these two inequalities implies failure of the LHS model, and therefore steering of A by B. The violation will also imply an EPR paradox in each case, because the inferred uncertainties represent the uncertainties of the "elements of reality," which exist to describe the local state of A, according to the EPR premises of local realism. These uncertainties are not compatible with the quantum uncertainty relation.

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