Minimum Power Routing for Multihop Cellular Networks

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Abstract—In multihop cellular networks, mobiles with no good path to any base station may instead relay their calls through other mobiles with better propagation conditions. This can improve coverage and capacity, and reduce the required total transmission power, but its effectiveness depends greatly on the routing strategy used. This paper investigates the minimum possible aggregate transmit power in the presence of interference in a single-cell multihop cellular network. The new concept of interference-sensitive link costs is introduced, and is shown to perform substantially better than routing based solely on path loss, which is optimal in the noise-limited case.

I. INTRODUCTION

Encouraged by the increasing popularity of ad-hoc networks, there has been interest in incorporating their multi-hop nature into cellular networks. This is the concept behind Opportunity Driven Multiple Access (ODMA) proposed in 3GPP [1], and is known within the mobile VCE in the UK as intelligent CDMA [2]. Providing a relaying capability in next-generation ad-hoc GSM (A-GSM) is also under study [3]. For data networks, multihop cellular networks have also been in [4].

The capacity, coverage and power requirements of a network depend heavily on the mutual interference between nodes. Thus, it is necessary to understand the properties of the topology which minimises the total transmit power in the presence of interference. Finding a suitable routing strategy is still an open problem. This problem is computationally intractable and heuristic algorithms are widely used. This paper takes the concept of joint power control and cell-site selection in spread-spectrum cellular networks [5] and combines it with a path-oriented minimum power routing algorithm. The resulting routing is then compared with the commonly used algorithm based on simply minimising path loss [6,7].

Multihop cellular networks and related routing techniques are reviewed in Section II. Section III presents a network model and defines the routing problem. Section IV investigates the power required to transmit a call over a link in the presence of low level interference. This gives rise to a new heuristic routing algorithm for minimising the total transmit power of a network. Simulation results verifying the effectiveness of this algorithm are presented in Section V.

II. MULTIHOP CELLULAR NETWORKS

Multihop cellular networks require less transmit power than single hop cellular networks, which increases battery life. Moreover, the coverage is improved by allowing a mobile in a propagation dead spot to be relayed by its neighbours. Thirdly, less total transmission power leads to less interference, potentially increasing the capacity. The above characteristics are well-suited to the concept of self-organising for future cellular networks [2], which minimises design costs.

Significant improvement in the required power transmission and coverage are demonstrated in [6], based on empirical path loss characteristics. In [8], increased capacity is reported for a time-slotted code division multiple access (CDMA) network. It is observed in [7] that the coverage of ODMA increases at much faster rate than that of single-hop case when the quality-of-service/traffic-load is reduced.

Although employing relaying reduces the average battery consumption, those mobiles which act as relays may actually be worse off, and the issue of fairness must be addressed. Moreover, more processing and signalling overhead will be required in the system. However, these problems can be addressed without removing multihop’s intrinsic benefits.

Multihop cellular networks rely heavily on good routing. In [6,7], the routing aims to minimise the sum of the path losses, subject to a constraint on the hop count. In a noise-limited environment, minimising the path loss is optimal. However, when the interference is significant, local congestion may occur which can increase the total transmit power unboundedly. In [8] the path with minimum aggregate power is selected from a pool of candidate paths, but it is not clear how the algorithm performs in the presence of interference.

One paper which does consider interference is [9]. It proposes a joint routing and channel assignment algorithm which aims to minimise the total power in the presence of interference. It assumes that each call’s path and channel are fixed after admission, but selects the channel and path for new calls in order to minimise the total power requirements. In contrast, the present paper seeks to rearrange a fixed set of calls in order to minimise the transmit power without changing the existing channel assignment. In [10], which appeared after the present research was completed, interference-based ODMA was proposed, but its interference-aware routing algorithm was not explicitly described.

III. MODEL AND PROBLEM DEFINITION

This work investigates the optimal and centralised routing algorithm for an uplink connection in a single-cell CDMA multihop network. There are \( M \) mobile stations, each of which must establish a path to a single base station \( B \). Fig. 1 shows a snapshot at a particular instance of multihop arrangement. Let traffic destined for the base station be called “uplink” traffic. Because a relaying mobile must simultaneously transmit and receive uplink traffic, the uplink requires at least two channels, either time-slots or frequency bands. Some mobiles must transmit on the first and receive on the second, and for some this is reversed. This classification is shown in Fig. 1 by black and white nodes. The base station itself is able to receive connections simultaneously using
Let $V$ be the set of all nodes in the network, where $b_1$ and $b_2$ are the two received channels at the base station, and $m_i, i = 1, \ldots, M$ are the $M$ mobile stations with their respective received channels.

Mobile station $m_i$ transmits at power $q_i$, and is received at node $j$ at strength $q_i \Gamma[i,j]$, where $\Gamma[i,j]$ is the path gain between nodes $i$ and $j$. Note that $\Gamma[i,j] = 0$ if nodes $i$ and $j$ have the same colour; that is, the channels are orthogonal. Receiver $j$ is subject to thermal noise of power $\eta W$, where $W$ is the bandwidth, and $\eta$ is the noise power spectral density. In Fig. 1, $m_7$'s signal is received at $b_2$, with thermal noise and interference from $m_1$, $m_2$, $m_6$, $m_9$, and $m_{10}$.

Let $r_i$ be the intended receiving node to which node $i$ is transmitting. The vector $r$ specifies the route of all calls. The carrier to interference density ratio for node $i$ is then

$$CIR_i(r) = \frac{W q_i \Gamma[i,r_i]}{\sum_{j=1}^M q_i \Gamma[j,r_i] + \eta[r_i] W},$$

where $\eta[r_i]$ is the thermal noise density at $r_i$. $CIR_i$ is given in Hz to show the effective bandwidth or quality of service achieved by node $i$.

It is assumed that each mobile station must transmit data to the base station at the same rate, requiring $CIR_i$. If node $i$ carries aggregate traffic from $n_i$ sources, its required $CIR$ becomes $CIR_i = n_i \alpha$, due to the reduced spreading factor. No constraints are currently imposed on the number of hops, the number of calls a mobile may relay, or resources available in each node. This is also illustrated in Fig. 1.

Given that the quality of service requirement of each user is $\alpha$, then the allocation of power $q_i$ to solve the $M$ linear equations implied by $CIR_i = n_i \alpha$ is given by [5]

$$q_i = (I - A)^{-1} b,$$

where $I$ is the $M \times M$ identity matrix, $A$ is the $M \times M$ normalised path gain matrix defined by

$$A_{ij}(r) = \begin{cases} n_i \alpha \Gamma[i,j,r_i] \frac{W}{\Gamma[i,r_i]} & \text{when } j \neq i, \\ 0 & \text{when } j = i \end{cases}$$

and $b$ is the $M \times 1$ vector defined by

$$b_i(r) = \frac{\eta[r_i] \eta \alpha}{\Gamma[i,r_i]}.$$

The vector $q_i$ specifies the power allocated to each node $i$ for one particular routing arrangement. The set of power allocation is feasible ($q_i > 0, \forall i$) if the dominant positive eigenvalue of matrix $A$ is less than 1.

The objective is then to find a path, $P_i$, for each user, $i$, minimising the cost, defined as the total transmitted power

$$Q = \sum_i q_i.$$
Recall that \( n_i \) calls are transmitted from node \( i \) to node \( r_i \), and let the path of a mobile, \( P \), be the set of links over which its data is carried. The total transmitted power in the network is then

\[
Q(r) = \sum_i q_i(r) = \sum_i n_i \alpha_i \eta_i / \Gamma[i, r_i]
\]

where \( \Delta q_{ir} = q_i / r_i \) is the transmit power from node \( i \) per connection carried.

Thus a standard shortest-path algorithm, such as Dijkstra’s algorithm, can be used with costs \( \Delta q_{lr} \). Moreover, if the thermal noise, \( \eta_i \), is assumed to be equal at all nodes, then the shortest path is simply that which minimises the sum of the transmission losses, \( 1 / \Gamma[i, j] \), over all links \((i, j)\) in the path. Note that these link costs are symmetric: \( C_{ij} = C_{ji} \).

**B. Low interference: New paths**

Although the “least loss” path requires the minimum transmit power when there is no interference, it is often far from optimal in the presence of interference; it often requires more power than connecting all mobiles directly to the base station, and is sometimes even infeasible. When interference is not negligible, the change of power caused by a link carrying one extra call is not as simple as (9). This is because each transmitter adjusts its power in accordance to the receiving end’s interference level. From Fig. 2, it can be seen that the change in \( q_1 \) and \( q_2 \) depends on interference levels \( I_2 \) and \( I_1 \), which themselves depend on how other nodes react to the change in \( q_1 \) and \( q_2 \). For illustration, consider Fig. 1, and let \( m_5 \) transmit an extra call to be relayed by \( m_7 \). This extra call requires \( m_5 \) and \( m_7 \) to increase their powers, \( q_5 \) and \( q_7 \). The increase of interference level will force the other mobiles to increase their powers in order to satisfy their CIRs. In the worst case, no feasible power allocation exists, and the network is heavily congested. The level of global congestion can be measured by the dominant positive eigenvalue of matrix \( A \) [11], but no information can be inferred about the level of congestion in a particular part of the network.

However, it will turn out that, given an existing configuration, it is meaningful to talk about the cost of an entire path. In this section, we assume that interference is non-zero but “small”, permitting a linearization of (2). Expanding (2), the change of total power by adding one call on a path \( P_i \) is

\[
\Delta q^i = (I - A)^{-1} \Delta b^i + (\Delta(I - A)^{-1})^i b
\]

Here \( \Delta b^i \) is a vector given by \( \alpha \eta_i / \Gamma[k, r_k] \) when \((k, r_k) \in P_i \) and 0 otherwise.

From (3), the change in matrix \( A \) when an additional call is added to path \( P_i \) is \( \Delta A^i \), with \( k \)th element

\[
\Delta A_{kk}^i(x) = \left\{ \begin{array}{ll} \frac{\alpha}{W} \Gamma[k, r_k] & (k, r_k) \in P_i \text{ and } k \neq l \\ 0 & \text{otherwise} \end{array} \right.
\]

Assuming that \( \Delta A^i \) is small compared with \( I - A \), the matrix \( (I - A)^{-1}) \Delta A^i (I - A)^{-1} \) can be linearized as

\[
\Delta((I - A)^{-1}) = (I - A)^{-1} \Delta A^i (I - A)^{-1}
\]

Further linearizing (11) gives

\[
\Delta q^i = (I - A)^{-1} \Delta b^i + (\Delta(I - A)^{-1})^i b
\]

\[
= \sum_{(i, j) \in P_i} c^i \left( \frac{\alpha \eta_j}{\Gamma[j, r_j]} + \Delta p^j \cdot d \right)
\]

\[
= \sum_{(i, j) \in P_i} \Delta q^j
\]

where \( c^i \) is the jth column of \( (I - A)^{-1} \), \( d = (I - A)^{-1} b \) and \( \Delta p^j \) is a vector whose \( l \)th element, \( l \neq j \), is

\[
\Delta p^j_l = \frac{\alpha}{W} \Gamma[j, r_j]
\]

Thus, when adding a new path to a system with low but non-zero interference, a standard shortest path algorithm can be used, with the cost of link \((i, j)\) being

\[
\hat{C}_{ij} = \sum_k \Delta q^j_k
\]

where

\[
\Delta q^j_k = c^j \left( \frac{\eta_j}{\Gamma[j, r_j]} + \Delta p^j \cdot d \right)
\]

(Note that, due to the presence of interference, the link costs are no longer symmetric: \( C_{jk} \neq C_{kj} \)).

However, this assumes that the total change in \( A \) due to the new path, \( \Delta A^i \), is small compared to \( A \). This approximation can be avoided using the algorithm presented in the following section.

**C. Re-routing**

In order to investigate the “optimal” routing for a multi-hop cellular network, it is useful to be able to take a particular network routing, with its associated interference levels, and produce a new
Routing with a lower total transmit power. This operation potentially requires substantial changes to the routing, with consequential changes in $A$. Thus the linearized costs of the previous section cannot be used for the entire re-routing operation. Instead, we propose a routing with a lower total transmit power. This operation potentially requires substantial changes to the routing, with consequential changes in $A$. Thus the linearized costs of the previous section cannot be used for the entire re-routing operation. Instead, we propose the On-the-Fly Dijkstra (OFD) algorithm, where the cost of each path is determined as it scans the nodes in the network.

Dijkstra's algorithm builds paths one link at a time. From a "candidate set", $D$, (which, in the wireless case, is the entire network) it builds up a "confirmed set", $F$, of nodes for which the shortest path to the destination is known. At each iteration, it transfers to $F$ the node in $D$ with the lowest path cost to the destination. This is repeated until all nodes are in the confirmed set, and all shortest paths are known.

The OFD algorithm starts with an initial routing configuration, but the two base station nodes (black and white) as the only members of the confirmed set. For each node $N \in D$, and for each node $c \in F$, the total transmit power of the network is calculated under the assumption that $N$ is re-routed via $c$, instead of $r_N$ of the original routing, that is, using

$$r^{N,c} = (r_1, \ldots, r_{N-1}, c, r_{N+1}, \ldots, r_M)^T. \quad (18)$$

If any of the resulting total transmit powers is lower than the current value, $Q(r^{N,c}) < Q(r)$, then the associated node, $N$ is added to the confirmed set. If none of the connections results in a reduction in transmit power, then a decision must be made between those $N$ for which the change is zero, namely those $N$ for which $r_N \in F$. These nodes are evaluated by how sensitive the total transmit power, $Q$, is to their load, $n_N$. For each $N \in D$ such that $r_N \in F$, the total transmit power is evaluated with load $n_N + 1$ on node $N$, and the node with the lowest resulting power is added to the confirmed set. This can be expressed in Algorithm 1.

This procedure either strictly reduces the total transmit power, or leaves the routing unchanged. Thus repeated application is guaranteed to converge. However, it will only converge to a local optimum.

The algorithm is centralised in that the path gains connecting every pair of nodes are known. Furthermore, path gains are assumed not to vary. By considering centralised routing we can gain insight into the absolute limits of the multi-hop approach. A real routing implementation will necessarily be distributed and its performance in terms of the total transmitted power is bounded by that of centralised routing.

V. SIMULATION

The ability of OFD to improve the current routing was evaluated by static simulations. Each snapshot consisted of 12 mobiles distributed uniformly on a disc around a single base station. The load was varied by changing the data rate, which was common to all mobiles. The path gain between nodes was

$$\Gamma[i, j] = r^{-1} f,$$

where $r$ is the distance separating them, and $f$ is log-normal shadowing with standard deviation 8 dB. To model the fact that mobile stations have simpler decoding circuitry, they had a target $E_b/N_0$ of 9 dB, compared with 6 dB for the base station. The spreading bandwidth was $W = 1.25$ MHz and the data rate was varied from 0.26 to 25.6 Kbps.

For each set of path gains, the total transmit power was calculated under each of the multi-hop routing strategies, and normalised by the power required to connect all mobiles to the base station directly. Due to the popularity of least-loss routing, we attempted to use it as a benchmark. However, as shown in Fig. 3, the least-loss route often requires more power than direct connection, and is often even infeasible due to the extra interference created by the relays. (Note that any non-zero probability of an infeasible configuration causes the average power to be infinite.) This problem was overcome by allowing each mobile to connect either to the relay on its least-loss path or to the base station. An exhaustive search of the $2^{12}$ possible combinations was performed, and the least-power configuration was termed the LLD configuration. The routing algorithms tested were: LLD, OFD/LLD (start from LLD and repeatedly apply OFD until convergence), OFD/direct (start with all mobiles connected to the base station and repeatedly apply OFD) and OFD/incremental (add mobiles one at a time, with OFD applied repeatedly between each addition).

To minimise the choice of channel allocation, only two channels were used, and OFD was not permitted to change the allocation, so that the gains reported here are conservative. For LLD, channels were allocated greedily to balance the number of calls reaching the base station on each channel. For OFD/direct, a maximum cut problem was found (see [12]), and the number of calls on each channel was balanced over the network.

The power reductions averaged over 5000–10000 independent snapshots are shown in Fig. 4. As the load (interference) increases, the total power will increase in both single- and multi-hop cases. It can be seen that LLD performs well only at very low load, i.e., in the noise-limited case. As the load increases, LLD's performance drops. This is because LLD only attempts to re-direct calls...
to the base station, and it doesn't re-organise the tail of its least-lost sub-trees. LLD's performance improves only when the load approaches the capacity of the single-hop system, where power warfare occurs in the single-hop case [5]. The performance of LLD can be further improved by OFD/LLD over the whole range of loads. The benefit of OFD is further shown by OFD/LLD-LLD, and even at very low interference OFD already provides improvement at a faster rate. However, this rate is restricted at very high interference.

The computational complexity of LLD is exponential in network size, which motivates the use of OFD with other initial configurations. For moderate loads, OFD/direct achieves a power reduction better than LLD. However, for high data rates, the performance drops significantly. This is because OFD gets trapped in local minima, where many mobiles are still connected directly to the base station and compete for high powers. This effect is magnified by the higher $E_b/N_0$ target at the mobile station receivers, but still occurs even with equal sensitivities. These local optima may be escaped by, for example, starting from low $E_b/N_0$ requirements and gradually increasing them to the true hardware requirements.

Under high interference, $A$ in (3) has an eigenvalue close to 1, which means that the power control algorithm converges very slowly. Thus, when incorporating the dynamics of power control into routing algorithm, one may allow to re-route connection before its target CIR is achieved (lower data rate). However, it is very unlikely that any operator will operate its single-hop networks near capacity where severe interference can cause instability. This was the motivation for OFD/incremental, which performs significantly better under high interference than OFD/direct.

The average (over all snapshots) of the number of hops of the longest path and the maximum number of calls relayed by a single mobile were both measured. Both of them decreased gradually as the load increased, with the exception of OFD/direct at very high load, where they dropped significantly as most calls remain connected to the base station. This suggests that at higher interference level, networks tend to re-organise so that traffic loads are balanced.

VI. CONCLUDING REMARKS

Least-lost based routing is optimal only in noise-limited environments. This paper has characterised the notion of link and path costs associated with wireless routing under interference. The proposed OFD routing scheme manages to re-configure an existing configuration into a much more power-efficient network.

The concept behind OFD can also be applied to constrained least-loss based routing algorithm; e.g., constraining the number of hops or number of calls a mobile may relay. Research is continuing into this and the implementation of OFD for multi-cell multi-hop cellular networks.

REFERENCES