EVIDENCE FOR A NEW ELLIPTICAL-GALAXY PARADIGM: SÉRSIC AND CORE GALAXIES

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ABSTRACT

We fitted the surface-brightness profiles of 21 elliptical galaxies using both the Sérsic function and a new empirical model that combines an inner power law with an outer Sérsic function. The profiles are combinations of deconvolved Hubble Space Telescope (HST) profiles from the literature and ellipse fits to the full WFPC2 mosaic images and thus span a radial range from ~0′02 to about twice the half-light radius. We are able to accurately fit the entire profiles using either the Sérsic function or our new model. In doing so, we demonstrate that most, if not all, so-called “power-law” galaxies are better described as “Sérsic galaxies”—they are well modeled by the three-parameter Sérsic profile into the limits of HST resolution—and that “core” galaxies are best understood as consisting of an outer Sérsic profile with an inner power-law cusp, which is a downward deviation from the inward extrapolation of the Sérsic profile. This definition of cores resolves ambiguities that result when the popular “Nuker law” is fitted to the profiles of ellipticals and bulges, particularly at lower luminosities. We also find that using the Nuker law to model core-galaxy nuclear profiles systematically overestimates the core radii by factors of 1.5–4.5 and underestimates the inner power-law slope by ~20%–40% or more.

Key words: galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: nuclei — galaxies: photometry — galaxies: structure

1. INTRODUCTION

The availability of high-resolution imaging with the Hubble Space Telescope (HST) has revolutionized the study of galaxy centers. Following up on early work by Crane et al. (1993), Kormendy et al. (1994), Grillmair et al. (1994), Jaffe et al. (1994), and Ferrarese et al. (1994), a series of papers by the “Nuker team” (Lauer et al. 1995; Byun et al. 1996; Gebhardt et al. 1996; Faber et al. 1997) presented a detailed study of the central regions of early-type galaxies (specifically, ellipticals and the bulges of spiral galaxies). They introduced a model for fitting the radial surface-brightness profiles: a double power-law with an adjustable transition region, dubbed the “Nuker law”:

\[ I(r) = I_0 2^{-(\beta-\gamma)/\alpha} (r/r_n)^{-\gamma} \left[ 1 + (r/r_n)^\gamma \right]^{-(\gamma-\beta)/\alpha}. \] (1)

The inner and outer power-law exponents are \( \gamma \) and \( \beta \), respectively; \( I_0 \) is the surface brightness at the core or “break” radius \( r_n \), and \( \alpha \) controls the sharpness of the transition between the two power laws (larger \( \alpha \) = sharper transition). They identified two distinct classes of galaxy centers: “power-law” galaxies, where the central surface brightness increases into the limit of resolution with something like a steep power-law profile; and “core” galaxies, where the luminosity profile turns over at a fairly sharp “break radius” into a shallower power law. Ferrarese et al. and Faber et al. found evidence that global parameters of early-type galaxies correlated with their nuclear profiles: core galaxies tend to have high luminosities, boxy isophotes, and pressure-supported kinematics, while power-law galaxies are typically lower luminosity and often have disky isophotes and rotationally supported kinematics.

The Nuker-law parameterization of galaxy centers has subsequently enjoyed a great deal of popularity, including extensive studies using WFPC2 and NICMOS (e.g., Rest et al. 2001; Quillen, Bower, & Stritzinger 2000; Ravindranath et al. 2001; Laine et al. 2003), and extensions to early- and late-type spirals (e.g., Carollo & Stiavelli 1998; Seigar et al. 2002). These more recent studies have, however, suggested that the clear core/power-law dichotomy found by the Nuker team may not be so clear after all. In addition, almost all the studies using HST data and Nuker-law fits have left unanswered a key question: how does the nuclear part of a bulge or elliptical, seemingly well fitted by a double power-law, connect to the outer profiles of such systems, which are generally well fitted by the Sérsic (1968) \( r^{1/n} \) function? In our first paper (Graham et al. 2003a, hereafter Paper I), we discussed some of the systematic problems and ambiguities that can arise when using a double power-law model to fit galaxy light profiles and suggested a new hypothesis and a new model that might resolve some of these problems. The hypothesis has two parts: first, that the nuclear (HST-resolved) profiles of most lower luminosity hot systems, including the power-law galaxies, are simply inward extensions of each galaxy’s outer profile, best modeled with a Sérsic function; second, that core galaxies are best modeled with our new function, an outer Sérsic function with a break to an inner power law. In this paper we make an empirical test of this proposed solution by modeling the entire light profiles of a sample of elliptical galaxies.

In the following we first review some of the problems stemming from the use of the Nuker law, including the problem of how best to identify genuine cores in galaxies (§2); readers familiar with these issues can probably skip this section. We
then discuss our sample selection, data reduction and analysis, and the source of the profiles used (§ 3). In § 4 we discuss the Sérsic model and our new model for core-galaxy profiles. Section 5 presents criteria for identifying core galaxies and for discriminating between core and Sérsic profiles. We also present the results of our fits to the galaxy profiles and compare their fidelity to the profiles with that of the Nuker-law fits. Some of the implications are discussed in § 6, and we conclude with a brief summary in § 7. Finally, several useful mathematical expressions related to our new model are presented in Appendix A.

2. SOME OUTSTANDING ISSUES

2.1. Relating Nuclear Surface Brightness Profiles to Outer Profiles

The progress engendered by the use of HST data and the Nuker law has tended to encourage a disconnect between the inner and outer regions of galaxies, which are studied separately and parameterized in different fashions. This is in part due to the fact that early HST studies using the first-generation Planetary Camera generally provided useful data only for $r \leq 10''$ (e.g., Lauer et al. 1995), so that only the nuclear region could be studied. It is also due to the fact that the Nuker law does not describe the light profiles outside this region well, even for “single-component” galaxies like ellipticals (e.g., Byun et al. 1996).

Meanwhile, there has been significant progress in understanding the luminosity structure outside the nuclear regions. These “global” surface brightness profiles are usually well described with Sérsic’s (1968) $r^{1/n}$ law, a generalization of de Vaucouleurs’ (1959) $r^{1/4}$ law. This has been shown to be true for both luminous ellipticals (e.g., Capaccioli 1987; Caon, Capaccioli, & D’Onofrio 1993; Graham et al. 1996) and dwarf ellipticals (e.g., Davies et al. 1988; Cellone, Forte, & Geisler 1994; Young & Currie 1994; Durrell 1997; Binggeli & Jerjen 1998; Graham & Guzmán 2003) and for the bulges of disk galaxies (Andredakis, Peletier, & Balcells 1995; Seigar & James 1998; Khosroshahi, Wadadekar, & Kembhavi 2000; Graham 2001; Balcells et al. 2003; MacArthur, Courteau, & Holtzman 2003). There is now good evidence that the shape of the overall surface brightness profile, as parameterized by the Sérsic index $n$, correlates with numerous (model-independent) elliptical and bulge properties: the total luminosity, the central surface brightness, the effective radius, and the central velocity dispersion (Graham, Trujillo, & Caon 2001; Möllenhoff & Heidt 2001; Graham 2002). It also correlates extremely well with the mass of central supermassive black holes (Graham et al. 2001b; Erwin, Caon, & Graham 2003). This clearly points to connections between the global distribution of stars in ellipticals and bulges and the properties of their nuclear regions, and makes it more important than ever to understand how the nuclear regions connect to the outer parts of galaxies.

2.2. The Ambiguity of Current Core and Power-Law Definitions

A second problem is the ambiguity of “core” versus “power-law” definitions and the apparent unraveling of the clear distinction between (high-luminosity) core and (lower luminosity) power-law galaxies reported by Faber et al. (1997), Rest et al. (2001) and Ravindranath et al. (2001)1 have found several examples of “intermediate” galaxies ($0.3 < \gamma < 0.5$; see, e.g., Fig. 3 of Ravindranath et al.); it is not clear where these galaxies fit into the core/power-law scheme. Taking a slightly different tack, Carollo et al. (1997) argued for a general trend of $\gamma$ versus absolute magnitude for ellipticals, with more luminous galaxies having shallower slopes: this roughly matches the trend found by Faber et al. 1997, but without splitting the galaxies into core and power-law categories. However, subsequent investigation of lower luminosity systems, particularly bulges in late-type galaxies and dwarf ellipticals, has shown a reversal of this trend: for low-luminosity systems, luminosity and inner power-law slope are anticorrelated (Stiavelli et al. 2001, especially their Fig. 4). This has also been portrayed as a dichotomy between more luminous “$R^{1/4}$” bulges, with high $\gamma$, and less luminous “exponential” bulges, which tend to have $\gamma < 0.3$ (Seigar et al. 2002).

To dramatize this problem, we plot $\gamma$ versus $M_B$ in Figure 1 for ellipticals spanning a wide range of absolute magnitudes, from the brightest core galaxies of Faber et al. (1997) down to the faint dwarf ellipticals of Stiavelli et al. (2001); a similar figure can be found in Graham & Guzmán (2003). We indicate the boundaries for core and power-law galaxies according to Faber et al. (1997); all galaxies plotted have well-resolved “cores” ($r_r > 0.16$). Two things stand out: first, there are numerous “intermediate” objects, so that the rather clear distinction reported by Faber et al.—that systems with small $\gamma$ are high luminosity, while systems with large $\gamma$ are lower luminosity—has become murky. Second, if we apply the standard definition of a core, then fully 21 of the 25 dwarf ellipticals of Stiavelli et al. (2001) have cores! Similarly, 12 of 38 spiral bulges (not plotted) studied in the optical by Carollo & Stiavelli (1998) and 10 of 45 bulges studied in the near-IR by Seigar et al. (2002) meet the standard criteria for

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1 Available at http://www.ociw.edu/ociw/symposia/series/symposium1/proceedings.html.

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*Fig. 1.—Problem of how to identify “cores”: inner logarithmic slope $\gamma$, from Nuker-law fits to HST profiles, vs. absolute magnitude $M_B$ for dwarf ellipticals from Stiavelli et al. (2001, asterisks) and regular ellipticals from Faber et al. (1997, circles), Rest et al. (2001, boxes), and Ravindranath et al. (2001, triangles). Filled symbols are core galaxies and half-filled symbols are “intermediate” galaxies, according to the authors of each study; Stiavelli et al. do not make core/noncore classifications. Total $B$ magnitudes are from LEDA, distances are from Tonry et al. (2001) or LEDA (corrected for Virgo infall and assuming $H_0 = 75$ km s$^{-1}$ kpc$^{-1}$); for Virgo cluster galaxies without measured distances, we assume $D = 15.3$ Mpc (Freedman et al. 2001). Only galaxies with Nuker-fit break radii $r_B > 0.16$ are plotted, so all galaxies with $\gamma < 0.3$ (bottom dashed line) are “core” galaxies according to the standard definition (Lauer et al. 1995; Faber et al. 1997); galaxies with $\gamma > 0.5$ (top dashed line) are “power-law” galaxies in the same scheme.*
having cores. Either both low- and high-luminosity galaxies—but not intermediate-luminosity systems—have cores, or we need a less problematic way of identifying cores. As we showed in Paper I, this kind of ambiguity arises automatically if the surface-brightness profile is exponential, or nearly so (i.e., a Sérsic function with \( n \leq 2 \)); when plotted in log-log space—and when fitted with a double power law such as the Nuker law—such profiles will seem to have cores. Since dwarf ellipticals and the bulges of many galaxies have profiles that are well fitted by Sérsic functions with small \( n \) (see references above), this is clearly a concern. The argument that Sérsic profiles only apply to the outer parts of profiles (that is, outside the region typically imaged by \( HST \)) is not tenable. Geha, Guhathakurta, & van der Marel (2002) and Graham & Guzmán (2003) were able to fit the \( HST \) profiles of dwarf ellipticals using Sérsic profiles (plus optional nuclear components). In addition, Jerjen, Bingelli, & Freeman (2000) found that the fully resolved surface-brightness profiles of Local Group dwarf spheroidals—which they show to be primarily the low-luminosity extension of the dwarf ellipticals—are quite well fitted by Sérsic profiles (see also Caldwell 1999).

Since the Sérsic shape parameter \( n \) is correlated with luminosity (e.g., Caon et al. 1993; Jerjen et al. 2000; Graham & Guzmán 2003) and with central velocity dispersion (Graham et al. 2001a; Graham 2002), we have a natural explanation for the correlation between \( \gamma \) and luminosity: Sérsic profiles observed from the ground continue inward into the regions resolved by \( HST \), so that galaxies with larger \( n \) (higher luminosities) will have larger \( \gamma \). Figure 2 shows that this is supported by the Sérsic fits and \( \gamma \) measurements of Stiavelli et al. (2001): dwarf ellipticals with larger values of \( n \) have larger values of \( \gamma \), in line with what we expect from Sérsic profiles observed at small radii. In \( \S \) 5 we show that the inner regions of higher luminosity, power-law ellipticals (high \( \gamma \)) are well fitted by Sérsic functions with large \( n \) that simultaneously fit the outer profiles.

But where does that leave core galaxies? The results of Gebhardt et al. (1996) and Faber et al. (1997) strongly suggest that the low-\( \gamma \) cores identified in these galaxies are genuine, physically distinct structures; indeed, some of these cores were well-known from high-resolution, ground-based imaging (e.g., Kormendy 1985; Lauer 1985; see the discussion in Lauer et al. 1995). The outer profiles of high-luminosity ellipticals, those most likely to have such cores, have large values of \( n \), so the inner slope \( \gamma \) should be large, the opposite of what is observed. This means that cores in bright ellipticals are clear deviations from the outer (Sérsic) profiles and suggest a more natural way of identifying cores: a downward deviation, with shallow logarithmic slope, from a galaxy’s outer Sérsic profile. This would resolve the ambiguity we noted above: illusory “cores” in low-luminosity systems (produced by fitting a double-power law to low-\( n \) Sérsic profiles) cannot be confused with true cores in high-luminosity systems. In \( \S \) 5 we show that this is indeed a viable approach: the complete profiles of high-luminosity core galaxies are not well fitted by a single Sérsic profile, but are well fitted by our new model, which joins a single, inner power-law profile to an outer Sérsic profile.

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\( ^2 \) Note that these authors do not classify centers into core/power-law categories and so do not actually label these “core” galaxies.

\( ^3 \) Eq. (A15) shows the relation between \( \gamma \) and \( n \) for a Sérsic profile.

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3. SAMPLE SELECTION, DATA REDUCTION, AND GENERATION OF PROFILES

3.1. Sample Selection

For this study, we needed a set of galaxies with \( HST \) observations of their central regions, as well as observations of the outer parts of the galaxies. Ideally, we want to compare our results with those of previous studies that used Nuker-law fits to analyze and classify the galaxies. This drove us to concentrate on the two largest \( HST \) studies of early-type galaxies: the WF/PC1 study of the Nuker team (Lauer et al. 1995; Byun et al. 1996) and Rest et al. (2001), which used WFPC2. In both cases the authors presented deconvolved profiles derived from ellipse fits to the Planetary Camera chips; Rest et al. (2001) also present values at very small radii derived directly from individual pixel values. Since these are the data that the Nuker team and Rest et al. use for their Nuker-law fits and classifications, it made sense for us to use them as well.

The problem then became finding suitable profiles for the galaxies outside the region imaged by the PC chips (\( r > 20'' \)). To minimize problems that might arise from combining profiles from different filters, we needed \( V \)-band images to go with the F555W profiles from Lauer et al. (1995) and \( R \)-band images to go with the F702W profiles from Rest et al. (2001). We also wanted images with fairly high resolution, to avoid any possible changes in curvature induced by trying to match ground-based profiles with poor seeing to the high-resolution \( HST \) images. The simplest solution to both of these requirements was to use \( HST \) images—in particular, WFPC2 images obtained using the same filters. Although the WFPC2 array is missing almost a quarter of its field, the overall field of view is \( \approx 2/6 \), which is sufficient to cover smaller galaxies; for larger galaxies, we can still sample most of the profile with the ellipse fits. In addition, the very low background in \( HST \) images means that we are less vulnerable to sky subtraction errors, which can affect the outer profiles. In practice, we found the following restrictions worked best: major axis <4'' and minor axis <3'', using the \( \mu_B = 25 \) dimensions from de Vaucouleurs et al. (1991, hereafter RC3).

The decision to use \( HST \) images makes the match with the inner profiles of Rest et al. (2001) particularly good: it means that we are using the exact same F702W images they used. For

![Fig. 2.—Inner logarithmic slope (\( \gamma \)) (from Nuker-law fits, averaged over \( r > 0''1--0''5 \)) vs. the Sérsic index \( n \) for the dwarf ellipticals of Stiavelli et al. (2001). Also plotted are curves showing the logarithmic slope of the Sérsic function at different fractions of the half-light radius (0.01, 0.05, and 0.1 \( r_h \)), derived using eq. (A15).](image-url)
the F555W profiles of the Nuker team, we searched the HST archive for WFPC2 images in the same filter (the F555W filters of the two cameras are not precisely identical, but the differences are too small to matter). There were somewhat fewer of these, so most of the galaxies we analyze are from the Rest et al. sample.

Finally, we decided to examine only elliptical galaxies. Although the bulges of disk galaxies are known to be well fitted by the Sérsic model, extracting the actual bulge profile means making bulge-disk decompositions. While not a significant problem for some galaxies, it does add some uncertainty, since we could end up fitting a one-dimensional profile with as many as eight free parameters (disk scale length and central surface brightness plus five or six parameters for our new model). In the future we do plan to analyze the bulges of disk galaxies using our new model, but for the purposes of this study we wanted to simplify matters and eliminate as much ambiguity as possible.

Thus, we selected only elliptical galaxies from the samples of the Nuker team and Rest et al. (2001). This meant not just selecting those galaxies classified as elliptical, but also ensuring that they were, in fact, true ellipticals with no significant disk component. A number of nominal E galaxies showed signs of having significant outer disks, suggesting that they may well be misclassified E/S0 or S0 galaxies. Our criteria included kinematic evidence from the literature, ellipse fits to the WFPC2 mosaic images, bulge+disk decompositions using the extranuclear ($r > 1''$) part of the profiles and the presence of substructures such as rings and bars, which are evidence for disks massive enough to be self-gravitating. Appendix C discusses rejected galaxies on a case-by-case basis. The remaining 21 galaxies, which we judged to be bona fide ellipticals, are listed in Table 1.

The angular size limits and the nature of the previous samples we draw on mean that the galaxies in Table 1 span a limited range in absolute magnitude. Happily, this narrow magnitude range ends up bracketing the overlap between core and power-law galaxies, and we have roughly equal numbers of each.

### 3.2. Data Reduction and Profile Matching

The WFPC2 images were retrieved from the HST archive with standard on-the-fly calibration. Multiple exposures were combined using the CRREJ task within IRAF. Alignment of different exposures was checked using coordinates of bright stars and galaxy nuclei; if the offset was ≤0.2 pixels in the PC chip, then the images were combined without shifting. (Since we use the published profiles of Lauer et al. 1995 and Rest et al. 2001 for $r \leq 10''$, we do not need highly accurate alignment.) We then made mosaic images from the combined exposures using the WMOSAIC task. Sky subtraction was based on the average of median values from several 10 × 10 pixel

### TABLE 1

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Type</th>
<th>$B_T$</th>
<th>$M_B$</th>
<th>Distance</th>
<th>Source</th>
<th>$V_{vir}$</th>
<th>Innermost Data</th>
<th>Profile Type</th>
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<td>24.1</td>
<td>3</td>
<td>1232</td>
<td>0.25/29.2</td>
<td>155 \</td>
</tr>
<tr>
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<td>E4</td>
<td>11.87</td>
<td>−21.36</td>
<td>44.3</td>
<td>3</td>
<td>3800</td>
<td>0.09/19.3</td>
<td>243 \</td>
</tr>
<tr>
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<td>E0−1</td>
<td>12.86</td>
<td>−18.32</td>
<td>17.2</td>
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<td>768</td>
<td>0.13/10.8</td>
<td>106 \</td>
</tr>
<tr>
<td>NGC 5845</td>
<td>E</td>
<td>13.24</td>
<td>−18.83</td>
<td>25.9</td>
<td>3</td>
<td>1634</td>
<td>0.02/2.8</td>
<td>244 \</td>
</tr>
</tbody>
</table>

The parameters have the same general meaning as in the Sérsic or Nuker laws: the break radius $r_b$ is the point at which the profile changes from one regime to another, $\gamma$ is the slope of the inner power-law region, $I_b$ is the intensity at the break radius, $\alpha$ controls the sharpness of the transition between the cusp and the outer Sérsic profile, $r_e$ is the effective radius of the profile, and $n$ is the shape parameter of the outer Sérsic part. The quantity $b$ is a function of the parameters $\alpha$, $r_b/r_e$, $\gamma$, and $n$, and is defined in such a way that $r_e$ becomes the radius enclosing half the light of the galaxy model (see Appendix A). If $\alpha \to \infty$, then the transition from Sérsic profile to power law at $r_b$ is infinitely sharp, with no transition region. In this limiting case the model can be written as

$$I(r) = I_b[(r_b/r)^\gamma u(r_b - r) + e^{b(r_b/r)^{\alpha}} e^{-b(r_b/r)^{\alpha}} u(r - r)],$$

with $u(x - a)$ being the Heaviside step function. Equation (5) can also be approximated using equation (3) with $\alpha \gtrsim 100$. Carollo & Stiavelli (1998) introduced a more limited version of equation (3), with a nonadjustable transition region and an exponential instead of the Sérsic outer region. (They used it to model—generally without success—the profiles of low-luminosity, “exponential” bulges with nuclear excesses, rather than those of the higher luminosity ellipticals which typically have cores.)

For the $\alpha \to \infty$ case, the relation between the intensity at the effective radius $r_e$ and the intensity at the break radius $r_b$, assuming that $r_e > r_b$, is given by

$$I(r_e) = I_b \exp\left[b\left((r_b/r_e)^{1/n} - 1\right)\right]$$

or, equivalently,

$$\mu_e = \mu_b - 2.5 b \left((r_b/r_e)^{1/n} - 1\right) \log e.$$
the sharp-transition model, eq. [5]). The Nuker law requires low values of \(\alpha\), for both core and power-law galaxies, because this is the only way to create the significant curvature needed to reproduce the observed curvature of galaxy profiles. But since the Sérsic part of our profile already models that curvature, we do not automatically need a low-\(\alpha\) transition. There are additionally two mathematical reasons for preferring the sharp-transition model. First, it reduces the number of free parameters in the model to five. Second, a smooth transition (low \(\alpha\)) distorts the meaning of the other parameters, so that, for example, the logarithmic slope of the inner profile is not equal to \(\gamma\) except at very small radii (as discussed in § 2).

Thus, we use both equations (3) and (5) to model galaxy profiles. Our hope, from the standpoint of simplicity and more transparent meaning for the model parameters, is that the sharp-transition model will be sufficient for core galaxies; as we show in § 5.3, this appears to be the case.

5. FITS TO GALAXY PROFILES

5.1. Fitting Techniques and Comparisons with Previous Fits

We fitted various models to the profiles using two standard nonlinear least-squares techniques: the downhill simplex (“amoeba”) method, and the Levenberg-Marquardt method (see, e.g., Press et al. 1992); many of the profiles were also fitted using a quasi-Newton algorithm (Kahaner, Moler, & Nash 1989). This went some way toward ensuring that our results were not dependent on the peculiarities of a single method or its implementation. In general, we found excellent agreement between fits obtained with the three methods. We also tried a variety of starting parameters, to ensure that our fits did not get trapped in local \(\chi^2\) minima. Following Byun et al. (1996), we weighted all points equally.

One test of our fitting methods is to see how well we reproduce the original Nuker-law fits of Byun et al. (1996) and Rest et al. (2001), if we restrict the radial range to that of the published PC profiles. In general, we did fairly well at this. There are minor differences between our Nuker-law fits and those of Byun et al. (typically only 10%–20% in parameter values) because the latter performed their fits to the equivalent radius \(r_{eq} = (ab)^{1/2}\) profiles, rather than to the major-axis profiles, as we do. They also used the (unpublished) cumulative \(r \leq 0.1\) flux as an additional constraint on the fits in some cases.

We found similarly good agreement with the original Rest et al. fits for about two-thirds of the galaxies drawn from their sample; but more significant differences exist for the remainder. There are two probable reasons for this. First, Rest et al. used a somewhat complex scheme of weighting the data points by the errors, while we weight all points equally. Second, their deconvolved profiles are often not evenly sampled in logarithmic radius; this can have the effect of giving more weight to points at smaller radii. For example, we get a much closer match to their Nuker-law fit for NGC 5576 if we fit to \(a \leq 5\)” in our combined profile, instead of \(a \leq 16\)” since there are fewer data points in their deconvolved profile beyond \(a = 5\)” (our combined profiles have had any such gaps filled in with points from the ellipse fits to the mosaic image to produce more evenly sampled profiles). This dependence on the radial weighting is probably a manifestation of the general radial sensitivity of Nuker-law fits (Papers I and III), a conclusion supported by the fact that when our fits differ significantly from those of Rest et al., our \(r_{eq}\) values are always larger.

5.2. Distinguishing Core from Sérsic Profiles

The Nuker team devised a simple set of criteria for separating core from power-law galaxies based on fitting profiles with the Nuker law (Lauer et al. 1995; Faber et al. 1997): if the Nuker-law break radius was large enough to be well-resolved (\(r_b \geq 0.16\)) and the inner power-law slope was sufficiently flat (\(\gamma \leq 0.3\)), then the galaxy was considered to have a core; otherwise, it was classed as power-law (or possibly as “intermediate”; e.g., Rest et al. 2001; Ravindranath et al. 2001; Laine et al. 2003).

Our approach is somewhat different: we want to determine when a galaxy profile is best fitted by one of two profiles, Sérsic or core-Sérsic, and—something that is in principle a separate issue—whether the galaxy has a core or not. Which model provides a better fit can be determined by comparing reduced \(\chi^2\) values. Galaxies that are well fitted with the Sérsic profile do not, by our definition, have cores. However, just getting a significantly better fit with the core-Sérsic model does not necessarily indicate a core. For example, a bright nuclear disk could add a distinct break to an underlying Sérsic profile; the composite would then be better fit by the core-Sérsic model, even though the overall elliptical/bulge profile was still Sérsic. As suggested in Paper I, we define a “core” as a downward deviation from the inward extrapolation of the outer (Sérsic) profile. Examples can be seen in Figures 3 and 4.

After some experimentation, we settled on the following criteria for clearly identifying core galaxies:

1. Qualitative identification of cores: attempting to fit an idealized core galaxy with a Sérsic profile produces a characteristic pattern in the residuals (Fig. 3). By fitting all galaxy profiles with the Sérsic model and examining the residuals, we can qualitatively identify core galaxies.

2. Significantly better fitted with core-Sérsic (CS) than with Sérsic models: \(\chi^2_{CS}(\text{Sérsic}) > 2\chi^2_{(CS)}(\text{CS})\) indicates that the core-Sérsic fit is clearly better, while \(\chi^2_{CS}(\text{Sérsic}) \leq 1.2\chi^2_{(CS)}(\text{CS})\) indicates the Sérsic profile is good enough. Intermediate ratios are ambiguous cases, which we discuss further below.

3. Potential cores must be both well resolved and represented by enough data points. Cases where the core-Sérsic break radius is greater than the innermost data point are potentially non-Sérsic profiles, but if the power-law regime is defined by only one or two data points, then its reality is dubious (and the inner slope \(\gamma\) will be poorly defined). Thus, for unambiguous core detection we require \(r_2 > r_0\), where \(r_2\) is the second innermost data point in the profile.

4. Finally, for a true core profile, we require that the power-law slope be consistently less than the logarithmic slope of the Sérsic fit inside break radius.8

Noncore galaxies can then be divided into two classes: pure Sérsic profiles and problematic cases, the latter usually due to a significant extra component such as a bright nuclear disk.

Figures 4 and 5 show the fits for core and Sérsic/ambiguous galaxies, respectively; Table 2 lists the parameters of the fits. The classifications are based on our fits, although, as we discuss below, we reproduce the core/power-law classifications of Byun et al. (1996) and Rest et al. (2001) almost perfectly. For each galaxy in the figures we show the best Sérsic and core-Sérsic fits to the entire profile. We also show the best Nuker-law fit to the inner profile obtained from the PC chip.

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8 This applies to the fitted data only; as \(r \to 0\), the Sérsic slope \(\to 0\) as well, but this happens well inside the resolution limit for all our galaxies.
We do this because we wish to compare how well a Sérsic or core-Sérsic fit to the entire profile manages to reproduce the inner profile, where the Nuker law has been used. The relative goodness of the fits is given in Table 2, where we list the reduced \( \chi^2 \) values \( \chi^2_{\nu} \) for the Sérsic and core-Sérsic fits, and in Table 3, where we give rms residuals for all three types of fit (Sérsic, core-Sérsic, and Nuker-law), evaluated in the inner (PC) region. Again, we do this so we can explicitly compare how well the global Sérsic or core-Sérsic fit does at reproducing the inner (HST-resolved) part of the profile.

5.3. Core Galaxies

Figure 4 shows the profiles and fits for the galaxies we classify as “core” or “possible core.” Notice that the pattern of the Sérsic-fit residuals for these profiles match the pattern in Figure 3: this is excellent (qualitative) evidence for genuine cores in these galaxies. As can be seen, fitting the profiles with the core-Sérsic model largely eliminates these residuals. Table 2 shows, in turn, that the core-Sérsic fits are significantly better, in a more quantitative, statistical sense, than the Sérsic fits for all but the two “possible core” galaxies: reduced \( \chi^2 \) values for Sérsic fits are larger by factors of \( \sim 3–15 \).

In general, we reproduce the core classifications of Rest et al. quite well, while finding that one of their “intermediate” galaxies (NGC 5557) is actually a core galaxy. We classify two galaxies, NGC 3613 and NGC 5077, as “possible core” galaxies. This is because, while the core-Sérsic fits are better than the Sérsic fits, they are not significantly so: \( \chi^2_{\nu}(\text{CS}) < 2\chi^2_{\nu}(\text{Sérsic}) \). The patterns of the Sérsic-fit residuals for these galaxies in Figure 4 do suggest possible core profiles, but again this is not strong enough to be convincing. In addition, the break radii from the core-Sérsic fits are near the inner limits of the data; for NGC 3613, \( r_B < 0'16 \), the nominal resolution limit of the Nuker team’s definition. For both galaxies, data at smaller radii are needed to really confirm (or deny) the apparent cores.\(^9\)

Table 2 includes the parameters and \( \chi^2 \) values for fits using both variants of the core-Sérsic model: free \( \alpha \) and \( \alpha = \infty \) (sharp transition between power-law and Sérsic regimes). By comparing the \( \chi^2 \) values for the core-galaxy and possible-core fits, we can see that in most cases the \( \alpha = \infty \) fit is only marginally better than the \( \alpha = \infty \) fit (see also col. [4] of Table 3). As we suggested in \( \S \) 4, the \( \alpha = \infty \) model generally provides just as good a fit as the free-\( \alpha \) version, while having one less free parameter and having parameters values (e.g., \( \gamma \)) that better describe the modeled profile.

There is only one galaxy (NGC 4168) where the free-\( \alpha \) fit is significantly different, in terms of parameter values, from the \( \alpha = \infty \) fit. We suspect this difference is probably due to the free-\( \alpha \) model being better able to fit noise or extra components in the profile, rather than being, e.g., an indication of a core with a genuinely broad transition region. First, there is filamentary dust in the nuclear region (Rest et al. 2001), which produces strong variations in the ellipse fits (Rest et al. and our Fig. 10). Second, the \( \alpha = \infty \) break radius (0"72, Table 2) matches the apparent break in the profile much better than the free-\( \alpha \) value (3"15), as can be seen in Figure 4. Third, the Sérsic index for the \( \alpha = \infty \) fit (\( n = 3.1 \)) is more reasonable than the free-\( \alpha \) value (\( n = 7.5 \)) for an intermediate-luminosity galaxy (see, e.g., Fig. 10 of Graham & Guzmán 2003). Finally, the rms residual values for both fits in the nuclear region (Table 3) are identical, which tells us that the free-\( \alpha \) fit does not provide a significantly better description of the core. For these reasons, we do not think the free-\( \alpha \) fit is genuinely better, and we prefer the \( \alpha = \infty \) fit for reasons of parsimony.

Finally, how do our core-Sérsic fits compare with Nuker-law fits in terms of reproducing the observed profiles? Table 3 compares rms residuals for the parts of the profile originally extracted from the PC chip of WF/PC1 or WFPC2 and fit with the Nuker law by Byun et al. (1996) and Rest et al. (2001). We remind the reader that the core-Sérsic fit is to the entire profile, while the Nuker-law fits are to the PC part of the profile only. Thus, the Nuker-law fit for NGC 3348, for example, is to semimajor axis \( a = 0'02–14'5 \), while the core-Sérsic fit(s) are to \( a = 0'02–78'5 \); but the rms residuals are determined for the same \( a = 0'02–14'5 \) region in both cases.

For the core galaxies, the core-Sérsic fit residuals in the PC region are never more than 20% larger than the Nuker-law residuals; the mean excess is only 3%, and for three of the seven galaxies, the core-Sérsic residuals are equal to or less than the Nuker-law residuals. This is rather astonishing, given

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\(^9\) Rest et al. (2001) noted an edge-on nuclear disk in the inner arc second of NGC 3613, which might explain some of the ambiguity if it is helping to mask a core or producing a corelike break in the profile.
that the core-Sérsic fit is constrained to fit the profiles out to \(~5\) times further in radius, while still having approximately the same number of parameters (exactly the same in the case of the \(\alpha = \infty\) core-Sérsic model). Casual inspection of Figure 4 shows that the Nuker-law fits become much worse than the core-Sérsic fits outside the PC part of the profile, as might be expected. We also note that the parameter \(r_f\) from our \(\alpha = \infty\) core-Sérsic fits is usually a closer match to the observed slope \((r_f',\) evaluated at \(r = 0.1\), from Rest et al. 2001) than is the Nuker-law parameter \(r_f;\) see Table 4.

5.4. Sérsic Galaxies

The remaining 12 galaxies (Figs. 5 and 6) are those for which there is no clear evidence for a core: the residuals of the Sérsic fits do not display the characteristic “core pattern” (Fig. 3), and the core-Sérsic fits are not significantly better in terms of \(\chi^2\). In fact, for seven of these 12 galaxies, one or both of the best core-Sérsic (\(\alpha\) free or \(\alpha = \infty\)) fits reproduces the best Sérsic fit: the \(n\) and \(r_e\) parameters are identical, and the core-Sérsic break radius \(r_b\) is less than the innermost data point. Core-Sérsic fits of this nature are clear evidence that these galaxies’ profiles are well described by pure Sérsic profiles. For another four of the galaxies, the \(n\) and \(r_e\) parameters differ by less than \(5\%\) between the core-Sérsic and Sérsic fits, and so the pure Sérsic profile is also preferred for reasons of simplicity.

All 12 of these galaxies were previously classified as power-law galaxies by Byun et al. (1996) or Rest et al. (2001) based on their Nuker-law fits. A comparison of the residuals (Table 3) shows that the Nuker law does fit the inner (PC) profiles slightly better, although, as Figures 5 and 6 show, the Nuker-law residuals are always worse—usually much worse—at larger radii. It is not too surprising that a fit using five parameters (the Nuker law) restricted to the inner 10”–17” does better in that region than a fit using only three parameters that

Fig. 4.—Fits to the surface brightness profiles (open circles) of core galaxies. For each galaxy, we show the best-fitting Sérsic (dashed line) and core-Sérsic (solid line; \(\alpha = \infty\) version) models. We also show the best-fitting Nuker-law profiles (dot-dashed line), fitted to the PC part of the profile only; the outer radius of the Nuker-law fits is marked by the vertical dotted line. Also shown are the residuals for each fit: Sérsic (open squares), core-Sérsic (filled circles), and Nuker (small diamonds). Finally, the break radii of the core-Sérsic (heavy arrow) and Nuker-law (light arrow) fits are indicated. In cases where the break radii of our Nuker-law fits differ significantly from the published fits of Rest et al. (2001), we indicate the published value.
also fits the profile out to 3–8 times further in radius. Nonetheless, for six of these galaxies, the (inner) Sérsic-fit rms residuals are less than 2 times the Nuker-law residuals, and for only one galaxy are the Sérsic residuals less than 3 times the Nuker-law residuals. As we discuss below, the strongest discrepancies are probably due to extra components such as nuclear disks.

There are five power-law galaxies where the Nuker fit is clearly better (in the inner region)—NGC 1426, 2634, 4458, 4478, and 5017. In four of these galaxies (NGC 1426, 4458, 4478, and 5845) there is clear evidence for a luminous nuclear disk (see Fig. 7 and the ellipse fits in Appendix B), with the break radius in the Nuker-law fits (and some of the core-Sérsic fits) occurring close to the point of maximum ellipticity associated with the nuclear disks. The distortions created by the nuclear disks in NGC 4458 and NGC 4478 are so strong—producing the largest residuals of any of the galaxies—that we do not consider the Sérsic fits to be reliable. A similarly strong nuclear disk (combined with a dust disk) is found in NGC 5845 (e.g., Quillen et al. 2000), so the Sérsic fit there may not be reliable either, although the Nuker-law fit is not dramatically better. There is evidence for a slight break in NGC 2634’s surface-brightness profile at $a \sim 2''$, though there is no accompanying signature in the ellipse fits—perhaps a face-on nuclear disk? NGC 5017 is also somewhat mysterious, but the fact that the core-Sérsic fits reproduce the Sérsic fit (Table 2) shows that this is not a core galaxy, and we tentatively include it with the Sérsic galaxies.

We note that the residuals for all of the fits to NGC 5796 are large, but this is clearly attributable to the noise in the profile at $a < 0''2$.

6. DISCUSSION

We conclude that most, if not all, “power-law” ellipticals are probably best understood as having Sérsic profiles—modulo extra components such as nuclear star clusters, nuclear disks, etc.—into the limits of resolution (or limits imposed by dust). As discussed in §2, this is consistent with an overall trend for
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Notes.—Structural parameters for fits to the major-axis profiles in our sample. For each galaxy, we list in the first row the best Sérsic fit ($n$, $r_e$, $I_e$), and in the next two rows the best core-Sérsic fits ($n$, $r_e$, $r_b$, $I_b$, $\gamma$, and $\alpha$; $\alpha = \infty$ is the sharp-transition version of the core-Sérsic model). When $r_b$ is listed in parentheses, then its value is less than the semimajor axis of the second innermost valid data point; consequently, the corresponding power-law region is poorly defined or meaningless. The criteria for assigning galaxies to the different categories (core, possible core, Sérsic) are discussed in the text. Col. (1): Galaxy name. Cols. (2)–(8): Best-fit parameters of the Sérsic and core-Sérsic models (eqs. [2], [3], [5]). The break radius $r_b$ and the effective radius $r_e$ are in arcseconds; $I_e$ and $I_b$ are in magnitudes per square arcsecond (observed values; no corrections for Galactic extinction or cosmological effects have been made). Col. (9): Reduced-$\chi^2$ values for the fits. Col. (10): Original HST inner profile classification from Nuker-law fits, from Lauer et al. (1995) or Rest et al. (2001); see Table 1. Col. (11): (1) inner parameters ($r_b$, $\chi^2_1$) dubious due to low value of $\chi^2_2$; (2) faint nuclear disk distorts profile; (3) bright nuclear disk distorts profile; (4) $r_b$ of indicated fit is between second and third data points of profile.

TABLE 3

Residuals of Fits in the Inner Region of Galaxy Profiles

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Profile Ranges</th>
<th>Sérsic rms (magnitudes)</th>
<th>CS rms (magnitudes)</th>
<th>Nuker-law rms (magnitudes)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Galaxies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N2986</td>
<td>0.02–14.4/76.9</td>
<td>0.19</td>
<td>0.054/0.054</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>N3348</td>
<td>0.02–14.5/78.5</td>
<td>0.14</td>
<td>0.040/0.042</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td>N4168</td>
<td>0.10–14.7/60.7</td>
<td>0.073</td>
<td>0.037/0.037</td>
<td>0.037</td>
<td></td>
</tr>
<tr>
<td>N4291</td>
<td>0.04–17.4/84.0</td>
<td>0.24</td>
<td>0.050/0.053</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>N5557</td>
<td>0.02–14.6/86.5</td>
<td>0.12</td>
<td>0.041/0.050</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>N5903</td>
<td>0.02–16.2/86.5</td>
<td>0.20</td>
<td>0.073/0.079</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td>N5982</td>
<td>0.03–17.0/79.2</td>
<td>0.15</td>
<td>0.030/0.037</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>Possible Core Galaxies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N3613</td>
<td>0.05–18.4/94.5</td>
<td>0.11</td>
<td>0.068/0.070</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>N5077</td>
<td>0.14–17.1/79.6</td>
<td>0.072</td>
<td>0.045/0.047</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>Sérsic Galaxies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N1426</td>
<td>0.35–10.2/81.6</td>
<td>0.041</td>
<td>0.030/0.031</td>
<td>0.015</td>
<td>1</td>
</tr>
<tr>
<td>N1700</td>
<td>0.13–10.2/62.5</td>
<td>0.061</td>
<td>0.061/0.061</td>
<td>0.028</td>
<td>2</td>
</tr>
<tr>
<td>N2634</td>
<td>0.10–13.7/55.5</td>
<td>0.066</td>
<td>0.046/0.046</td>
<td>0.027</td>
<td></td>
</tr>
<tr>
<td>N2872</td>
<td>0.39–14.6/49.3</td>
<td>0.045</td>
<td>0.045/0.045</td>
<td>0.028</td>
<td>2</td>
</tr>
<tr>
<td>N3078</td>
<td>0.63–16.7/79.2</td>
<td>0.025</td>
<td>0.025/0.025</td>
<td>0.015</td>
<td>2</td>
</tr>
<tr>
<td>N4458</td>
<td>0.18–14.5/68.2</td>
<td>0.16</td>
<td>0.16/0.16</td>
<td>0.045</td>
<td>1, 2</td>
</tr>
<tr>
<td>N4478</td>
<td>0.02–14.9/70.3</td>
<td>0.23</td>
<td>0.23/0.15</td>
<td>0.11</td>
<td>1, 3</td>
</tr>
<tr>
<td>N5017</td>
<td>0.33–15.2/55.5</td>
<td>0.080</td>
<td>0.080/0.080</td>
<td>0.027</td>
<td>2</td>
</tr>
<tr>
<td>N5576</td>
<td>0.02–16.0/77.5</td>
<td>0.073</td>
<td>0.063/0.064</td>
<td>0.046</td>
<td></td>
</tr>
<tr>
<td>N5796</td>
<td>0.02–12.7/79.7</td>
<td>0.15</td>
<td>0.15/0.15</td>
<td>0.14</td>
<td>3</td>
</tr>
<tr>
<td>N5831</td>
<td>0.02–14.9/68.3</td>
<td>0.061</td>
<td>0.057/0.061</td>
<td>0.053</td>
<td>3</td>
</tr>
<tr>
<td>N5845</td>
<td>0.02–10.2/39.0</td>
<td>0.097</td>
<td>0.060/0.062</td>
<td>0.064</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes.—Comparison of rms residuals for various fits in the inner region (defined as that region fit with the Nuker law for each galaxy in Byun et al. 1996 or Rest et al. 2001). The Nuker-law rms is from our fit to the corresponding region; the Sérsic and core-Sérsic (CS) rms are from our fits to the entire profile, with the residuals calculated in the inner region only. Col. (1): Galaxy name. Col. (2): Fitted regions of profile (semimajor axis, in arcseconds). The first range is the “inner region” (fitted with Nuker law), followed by outer limit of the Sérsic and core-Sérsic fits. Col. (3): rms residuals, in magnitudes, of Sérsic fit, calculated in Nuker-law fit region. Col. (4): Same as (3), but for the core-Sérsic fits—first number is for free-$\alpha$ version, second is for $\alpha = \infty$. Col. (5): rms residuals of Nuker-law fit. Col. (6) Notes: (1) nuclear disk; (2) both core-Sérsic fits reproduce Sérsic fit; (3) $\alpha = \infty$ core-Sérsic fit reproduces Sérsic fit.
elliptical galaxies: low- and intermediate-luminosity ellipticals have pure Sérsic profiles (plus optional nuclear disks, clusters, and point sources), and distinct cores appear in high-luminosity systems as deviations from the outer Sérsic profile. (Graham & Guzmán 2003 combine measurements for a large set of elliptical galaxies, including dwarf ellipticals, to make this argument in more detail.) Moreover, for power-law galaxies, we get excellent fits using a model with fewer parameters, all of which are physically meaningful (i.e., correlate with other galaxy parameters). These fits work for the entire profile, unlike the Nuker law, yet are as good a fit in the region where the Nuker law is usually used.

The term “power-law galaxy” is thus somewhat misleading, since it suggests that the nuclear profile is adequately described by a single power law, which is probably different from the outer profile. While this is an appealingly simple description for modeling purposes, our results strongly suggest that this is not accurate. Instead, elliptical galaxy profiles have logarithmic slopes that continuously decrease as $r \rightarrow 0$. Figure 11 of Lauer et al. (1995), which presents representative examples of “power-law” profiles, supports this argument: even the galaxy that is closest to a perfect power law, NGC 1700, shows a systematic deviation from a power law—steeper at larger radii, shallower at smaller radii—as expected for a Sérsic profile; see Figure 5. (This is not the case for the central cusps of core galaxies; their Fig. 7.)

The “intermediate” galaxies reported by Rest et al. (2001) and Ravindranath et al. (2001) are probably a consequence of Nuker-law fits applied to this overall elliptical-galaxy trend. Lower luminosity “intermediate” galaxies are most likely Sérsic galaxies with low values of $n$ (and hence $\gamma$) in the range 0.3–0.5; see Fig. 2). At higher luminosities core galaxies can appear to have $\gamma > 0.3$ if the core is not adequately resolved (either because of distance or inner truncation of the profile by, e.g., dust). (We do classify two galaxies in our sample as “possible core” galaxies, but these are clearly cases of inadequate resolution.)

Although we have not yet attempted to model the complete profiles of bulges, it is reasonable to extend our results to them. Baulells et al. (2003) have already done this for a sample of early-type bulges in the near-IR, using NICMOS data in conjunction with ground-based imaging. They find that the complete bulge profiles, after accounting for the presence of the outer disk, can be well modeled by Sérsic profiles, plus optional nuclear components (corresponding to, e.g., nuclear star clusters or point sources). This is in excellent agreement with our hypothesis that the profiles of lower luminosity ellipticals and bulges are fundamentally Sérsic profiles and promises to resolve a number of ambiguities and “dichotomies” reported in the literature. For example, Carollo et al. (1997) and Seigar et al. (2002) argue for a dichotomy between $R^{1/4}$ and exponential bulges, with the latter having low $\gamma$ in contrast to the high $\gamma$ of $R^{1/4}$ bulges and moderate-luminosity ellipticals. This is naturally explained if most bulges actually have Sérsic profiles (as is well supported by a number of studies) and if these Sérsic profiles extend into the nuclear region. The division between $R^{1/4}$ (Sérsic index $n = 4$) and exponential ($n = 1$) bulges is probably an artificial one, given that bulges in reality show a range of values of $n$. But as Paper I shows, bulges with larger $n$ will have higher values of $\gamma$ than bulges with low $n$. Thus, $R^{1/4}$ bulges (higher $n$) will exhibit larger values of $\gamma$ than “exponential” (lower $n$) bulges.

In retrospect, we can see that most of the early HST studies of galaxy centers, as well as some of the more recent ones (e.g., Rest et al. 2001; Ravindranath et al. 2001), have focused on relatively high-luminosity systems. These samples thus included a mix of Sérsic galaxies with high $n$ values and genuine core galaxies, making a distinction between core and “power-law” galaxies based purely on $\gamma$ feasible. More recent studies aimed at low-luminosity systems (e.g., Carollo & Stiavelli 1998; Stiavelli et al. 2001; Seigar et al. 2002) have since uncovered evidence for the low-$n$–low-$\gamma$, high-$n$–high-$\gamma$ trend that pure Sérsic profiles generate and thus show that discriminating core galaxies purely by $\gamma$ is problematic at best.

### 6.1. Core Identifications and Core Parameters

We find that most of the previously identified “core” galaxies in our sample do have distinct cores with shallow, power-law cusps. These cores stand out as downward deviations from the outer Sérsic profiles. Fitting with the core-Sérsic model provides a more natural, less ambiguous definition for “true” cores, without the possibility of misclassifying low-$n$ Sérsic profiles as cores. We are also able to reclassify one of the “intermediate” galaxies (NGC 5557) of Rest et al. (2001) as a core galaxy. The two ambiguous galaxies—NGC 3613 and NGC 5077—are simply cases where the apparent break radius

### Table 4: Comparison of Core-Sérsic and Nuker Parameters for Cores

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$I_0$(CS)</th>
<th>$r_0$(CS)</th>
<th>$\gamma$(CS)</th>
<th>$I_0$(Nuk)</th>
<th>$r_0$(Nuk)</th>
<th>$\gamma$(Nuk)</th>
<th>$\gamma'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N2986</td>
<td>15.5</td>
<td>0.69/97</td>
<td>0.25</td>
<td>16.1</td>
<td>1.24/174</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>N3348</td>
<td>15.6</td>
<td>0.25/70</td>
<td>0.16</td>
<td>16.0</td>
<td>0.99/198</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>N3613</td>
<td>14.7</td>
<td>0.15/21</td>
<td>0.09</td>
<td>15.1</td>
<td>0.34/48</td>
<td>0.04</td>
<td>0.17</td>
</tr>
<tr>
<td>N4168</td>
<td>16.7</td>
<td>0.72/108</td>
<td>0.22</td>
<td>17.5</td>
<td>2.02/303</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>N4291</td>
<td>14.5</td>
<td>0.37/47</td>
<td>0.14</td>
<td>15.1</td>
<td>0.60/76</td>
<td>0.00</td>
<td>0.13</td>
</tr>
<tr>
<td>N5077</td>
<td>15.2</td>
<td>0.36/62</td>
<td>0.29</td>
<td>16.5</td>
<td>1.61/279</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td>N5557</td>
<td>14.7</td>
<td>0.23/51</td>
<td>0.23</td>
<td>16.2</td>
<td>1.21/269</td>
<td>0.14</td>
<td>0.33</td>
</tr>
<tr>
<td>N5903</td>
<td>16.2</td>
<td>0.86/141</td>
<td>0.15</td>
<td>16.8</td>
<td>1.59/262</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>N5982</td>
<td>14.8</td>
<td>0.28/57</td>
<td>0.11</td>
<td>15.6</td>
<td>0.74/151</td>
<td>0.00</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Notes.—Comparison of core parameters obtained from core-Sérsic (CS) and Nuker-law (Nuk) fits to the core galaxies. The break radii $r_0$ are in arcseconds/parsecs; $R$-band brightness at the break radius is in magnitudes per square arcseconds. We use the $\alpha = \infty$ (sharp-transition) version of the core-Sérsic model for the CS values; the Nuker-law values and the slope at $r = 0.1''$ ($\gamma'$) are taken from the original fits in Rest et al. 2001.
Fig. 6.—Same as Fig. 5, but showing fits for galaxies with prominent nuclear disks.

Fig. 7.—Isophote contours (top) and unsharp masks (bottom) of PC images of NGC 4458 and NGC 4478, showing the prominent nuclear disks in each (see also the ellipse fits in Fig. 10). These nuclear disks introduce strong deviations from a pure Sérsic models in the surface brightness profiles. (Similar effects are produced by the nuclear disk in NGC 5845; see Quillen et al. 2000.)
is very close to the inner limits of the data. For NGC 3613, this is because the apparent core is close to the resolution limit (in fact, \( r_b \) from the core-Sérsic fits is less than 0.016 and thus smaller than the suggested resolution-based limit of Faber et al. 1997). For NGC 5077, on the other hand, Rest et al. (2001) clipped their data at \( r = 0.01 \) because of an apparent nuclear excess at smaller radii. A future fit including data at smaller radii and using an extra nuclear component to account for this excess may help determine whether NGC 5077 truly possesses a core.

While our overall agreement with the core/noncore classifications of Lauer et al. (1995) and Rest et al. (2001) is quite good for the galaxies we analyze, we find that Nuker-law fits systematically overestimate the size of the cores: our break radii are \( \sim 1.5–4.5 \) times smaller in size than the break radii from the published Nuker-law fits. Consequently, \( \mu_b \) values are brighter as well. We also find consistently higher values of \( \gamma \), though the difference is not as dramatic (see Table 4 and Fig. 8). This is in excellent agreement with the arguments of Papers I and III: all Nuker-law parameters are sensitive to the radial size of the region where the fit is made. All parameters of the Nuker model, including \( \gamma \) and \( r_b \), must be adjusted to fit both the core and the (non-power-law) part of the profile outside, with its intrinsic (Sérsic) curvature. Table 4 shows that, on average, the core-Sérsic values of \( \gamma \) match the observed core slope \( \gamma' \) (as determined by Rest et al. 2001) better than the Nuker-law values do.

The currently favored theory for core formation is the ejection of core stars by three-body encounters with a decaying black hole binary formed following a merger of two galaxies with central supermassive black holes. Various calculations (Ebisuzaki, Makino, & Okamura 1991; Quinlan & Hernquist 1997; Milosavljević & Merritt 2001) have estimated the stellar mass ejected during this process \( (M_{ej}) \), and generally find it to be \( \sim M_{BH} \), where \( M_{BH} \) is the mass of the resulting central black hole formed by the (assumed) coalescence of the binary. However, attempts to test these predictions by estimating \( M_{ej} \) from observed cores and comparing it with various estimates of \( M_{BH} \) consistently produce values of \( M_{ej} > M_{BH} \). Faber et al. (1997) found \( M_{ej} = 3.5–6.4 \) \( M_{BH} \); using more accurate estimates of \( M_{BH} \), Milosavljević & Merritt (2001) found \( M_{ej} \approx 1–20 \) \( M_{BH} \). Ravindranath, Ho, & Filippenko (2002) used the prescription for \( M_{ej} \) of Milosavljević & Merritt (2001) and a much larger data set; they found \( M_{ej} \approx 2–20 \) \( M_{BH} \) at the low-mass end \( (M_{BH} \sim 10^8 \) \( M_\odot \)) while at the high-mass end \( (M_{BH} \sim 10^9 \) \( M_\odot \)) \( M_{ej} \approx 6–25 \) \( M_{BH} \). Even considering only the galaxies with measured \( M_{BH} \), \( M_{ej}/M_{BH} \approx 4–13 \). Milosavljević & Merritt pointed out that the total ejected mass should increase with the number of mergers, but the observed ratios still seem high, particularly at the low-mass end, where there have presumably been fewer mergers.

All of the studies cited above used parameters from Nuker-law fits to estimate \( M_{ej} \). Since the estimated \( M_{ej} \) scales with \( r_b \) in the parameterization introduced by Milosavljević & Merritt (2001) and used by Ravindranath et al. (2002), \( M_{ej} \propto r_b \) overestimating \( r_b \) will naturally overestimate \( M_{ej} \). Thus, at least some of the discrepancy between observed and predicted \( M_{ej}/M_{BH} \) is probably due to the tendency of Nuker-law fits to overestimate \( r_b \), as we have found. Assuming that the core radii from core-Sérsic fits are typically \( \sim 2–4 \) times smaller than the Nuker-law values, as is the case for our sample, \( M_{ej}/M_{BH} \) values should go down by comparable factors, which would put them in better agreement with the theoretical predictions.

One of our core galaxies (NGC 4291) was noted by Ravindranath et al. (2001) for possibly having an isothermal core (with \( \gamma = 0 \)), on the basis of their Nuker-law fits to a NICMOS image. The Nuker-law fit in Rest et al. (2001) to the WFCPC profile also has \( \gamma = 0.0 \), which might seem to strengthen the case for an isothermal core. However, we find \( \gamma = 0.14 \) from our core-Sérsic fit, which agrees very well with \( \gamma' = 0.13 \), determined by Rest et al. So the core of NGC 4291 is probably not isothermal.

In Figure 9 we show the relation between the core and the global properties of the galaxies in our sample. We also indicate the upper limits on possible core radii for the Sérsic galaxies, based on the radii of the innermost valid data. For those galaxies where a clear core has been measured, we find that the relation between the break radius and the effective radius is approximately given by \( r_b = 0.014 r_e \). This is a factor of 2 smaller than the relation found by Faber et al. (1997), consistent with our finding that fitting with the Nuker law tends to overestimate core sizes.

There is a suggestion of a weak trend of \( r_b \), increasing with galaxy luminosity, which would be in agreement with what Faber et al. found (see also Laine et al. 2003), but for our sample this “trend” is anchored by only two points, so it is dubious. Unfortunately, the narrow magnitude range spanned by the core galaxies in our sample \( (\leq 1.5 \) mag) precludes a proper test of the magnitude-\( r_b \) relation reported Faber et al., which is based on galaxies spanning \( \pm 3 \) mag (and the composite trend in Fig. 9 of Laine et al. spans almost 5 mag). There is no clear magnitude-related trend in the ratio of our \( r_b \) measurements to the Nuker-law measurements, which suggests that the magnitude-\( r_b \) trend may be unaffected by changes in \( r_b \), except possibly in the scatter. However, a proper evaluation of how the magnitude-\( r_b \) relation is affected by better measurements of \( r_b \) must await core-Sérsic fits to a larger sample of core galaxies. There is no
evidence for a relationship between $n$ and $r_b$; this may be partly due to large uncertainties in $n$ (Caon et al. 1993 found typical errors of $\sim 25\%$ when fitting Sérsic profiles). Finally, we find no clear correlation between $\gamma$ and the global properties of the core galaxies analyzed. This is agreement with what previous studies have found for core galaxies (e.g., Rest et al. 2001, Fig. 7; Ravindranath et al. 2001, Fig. 3; Laine et al. 2003, Fig. 6; and the core galaxies in Fig. 1 of this paper).

6.2. Hidden Cores and the Core-Galaxy Fraction

An interesting point is to consider how well resolved the underlying profiles of the various galaxies actually are. In several cases Byun et al. (1996) and Rest et al. (2001) excluded points at small radii from their fits, usually because of the presence of significant nuclear dust or a distinct nuclear component (e.g., a nuclear point source). Thus, not all of the profiles take full advantage of HST resolution. While the nuclear components may include cases of nuclear star clusters, which make discussions of the underlying stellar profile ambiguous, the presence of dust means that some “power-law” (i.e., Sérsic-profile) galaxies could have hidden cores.

If we divide the sample into two groups—galaxies where the innermost valid data point is at $r < 15$ pc (spatially well-resolved centers); and galaxies where the innermost valid point is at $r > 15$ pc (less well-resolved centers)—we find that the less resolved galaxies are almost all\(^{10}\) well fitted using just the Sérsic model. This suggests that at least some of the Sérsic galaxies could have “hidden” cores. This is not a new argument, obviously, as many authors have pointed out that “power-law” galaxies could include unresolved cores—but it is interesting to consider how few of the Sérsic galaxies in our sample can really be declared free of HST-resolvable cores. Of the 21 galaxies, seven clearly have cores, two have possible cores (NGC 3613 and NGC 5077; see § 6.1), and only five (NGC 4478, 5576, 5796, 5831, and 5845) are clearly free of significant ($r_b > 5$ pc) cores.

So in the limited range of absolute magnitude spanned by our full sample ($-18.3 \lesssim M_B \lesssim -21.4$), 33% of the galaxies have unambiguous, HST-resolved cores; but this is clearly a lower limit. The core fraction rises to 43% if we include the two possible cases, and in principle could be as high as 76%. It is also interesting to note that we can see in the absolute magnitudes a hint of the well-known dichotomy between core and noncore galaxies (see, e.g., the discussion in Rest et al. 2001), even in our limited sample. This can be seen in Figure 9, where the five fully resolved Sérsic galaxies tend to be fainter than the core galaxies; a Kolmogorov-Smirnov test gives a 95% probability that the two groups of galaxies come from different parent luminosity distributions.

7. SUMMARY

We have successfully fitted the complete surface-brightness profiles of 19 out of 21 elliptical galaxies, from the HST-resolved central regions ($r \sim 0.02$) out to about twice the half-light radius, using either (1) a pure Sérsic profile or (2) a “core-Sérsic” model consisting of an outer Sérsic profile joined to an inner power-law core. The former fits correspond

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\(^{10}\) The exceptions are NGC 4168 (core) and NGC 5077 (possible core).
to so-called “power-law” galaxies, which are perhaps better described as “Sérsic galaxies,” and the latter correspond to core galaxies.

The combined use of these two models lets us address the following questions:

1. How can we relate the central, HST-resolved part of the galaxies’ surface-brightness profiles to the outer regions? We show that most power-law ellipticals are well described at all radii by the simple Sérsic law (modulo any nuclear disks, etc.). On the other hand, core galaxies are extremely well fitted with the core-Sérsic model. We find little need for a significant transition region between the outer (Sérsic) part of the core-Sérsic profile and the (power-law) core; any such transition region is small compared with the size of the core.

2. Is there a dichotomy in nuclear profiles between low- and high-luminosity bulges and ellipticals? Some recent HST studies have suggested that the apparent trend seen in intermediate- and high-luminosity bulges and ellipticals—cores with shallow logarithmic slopes in high-luminosity systems, steeper nuclear slopes in lower luminosity ("power-law") systems—breaks down at lower luminosities, because fainter bulges and dwarf ellipticals have shallow nuclear slopes. We show that the power-law galaxies in our sample have Sérsic profiles that extend into the limits of HST resolution, with $n \sim 4-6$; this naturally explains the steep nuclear slopes previously reported. When combined with the well-known correlation between $n$ and luminosity, we can see that (as argued by Graham & Guzmán 2003) the general trend is most likely one of pure Sérsic profiles (plus possible extra components, such as nuclear star clusters and disks) extending from low-luminosity systems with low-$n$ Sérsic profiles—and thus shallow nuclear slopes—to high-luminosity systems with high-$n$ profiles and steeper nuclear slopes. Only the high-luminosity core galaxies break the trend, because of the existence of the cores themselves.

3. How can we unambiguously identify cores in galaxy profiles? As we demonstrate, the traditional definition of cores using parameters from Nuker-law fits to galaxy profiles ($r_b \geq 0.16$ and $\gamma < 0.3$) leads to the real possibility of misclassifying galaxies with sufficiently shallow slopes (for example, exponential profiles) as core galaxies. We define core galaxies as those possessing a well-resolved downward deviation from the inward extrapolation of the outer (Sérsic) profile. This definition recovers previous core definitions for the high-luminosity ellipticals in our sample, but is immune to the danger of identifying exponential-like profiles as having cores.

4. How can we more accurately determine the structural properties of cores? As demonstrated in Paper I, the Nuker law requires a broad, smooth transition (low values of $\alpha$) between its two power-law regimes to fit the inner profiles of core and power-law galaxies, because this is the only way to reproduce the observed curvature of actual galaxy profiles. We find that this causes the core-size measurements (i.e., the break radius) to be overestimated by factors of 1.5–4.5 in comparison with the values derived by using the core-Sérsic model, which directly accounts for the intrinsic curvature of galaxy profiles. We also find that the logarithmic slope $\gamma$ of the observed core is more accurately recovered with the core-Sérsic model. Using the smaller values we find, especially for $r_b$, should bring estimates of the ejected stellar mass due to core formation more in line with theoretical predictions.

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APPENDIX A

SOME USEFUL MATHEMATICAL EXPRESSIONS RELATED TO THE CORE-SÉRIS MODEL

A1. THE RELATION BETWEEN CORE-SÉRIS AND SÉRIS EFFECTIVE RADII

In this section we want to prove the following identity:

$$b_n(1/r_\alpha)^{1/n} = b(1/r_e)^{1/n},$$

(A1)

where $r_\alpha$ is the effective radius of the Sérsic part of the core-Sérsic model, $r_e$ is the effective radius of the global core-Sérsic model, and $b_n$ and $b$ are the quantities introduced in order to give to $r_e$ in the Sérsic and core-Sérsic model, respectively, the meaning of effective radius.

_Demonstration._—Although the above relation can be proved for smooth transitions between the Sérsic regime and the power-law regime (i.e., $\alpha$ small), we will only show the demonstration for the sharpest transition case ($\alpha \rightarrow \infty$). The Sérsic part of the core-Sérsic model is described using the following law:

$$I(r) = I(0) \exp \left[ -b(r/r_\alpha)^{1/n} \right],$$

(A2)

with

$$I(0) = I_b \exp \left[ b(r_b/r_e)^{1/n} \right].$$

(A3)
The integrated luminosity out to a given radius for this model is given by

\[ L(r) = (2\pi n/b^2)r^2 I(0) \gamma \left[ 2n, b(r/r_e)^{1/n} \right], \]  

(A4)

with \( \gamma(a, x) \) being the incomplete gamma function. We can now determine the effective radius \( r_{es} \) for equation (A2) using the effective radius equation

\[ 2L(r_{es}) = L(\infty), \]  

(A5)

with \( L(\infty) \) being the total luminosity. For equation (A2), the effective radius equation becomes

\[ 2\gamma \left[ 2n, b(r_{es}/r_e)^{1/n} \right] = \Gamma(2n), \]  

(A6)

where \( \Gamma(a) \) is the complete gamma function. On the other hand, if we have a pure Sérsic law described by the index \( n \), the above equation is written as

\[ 2\gamma(2n, b_n) = \Gamma(2n). \]  

(A7)

It follows immediately that

\[ b_n = b(r_{es}/r_e)^{1/n}, \]  

(A8)

or, equivalently,

\[ b_n(1/r_{es})^{1/n} = b(1/r_e)^{1/n}, \]  

(A9)

as we wanted to show.

**A2. The Evaluation of \( b \) for the Core-Sérsic Model**

The quantity \( b \) is used in the Sérsic and core-Sérsic models to give \( r_e \) the meaning of effective radius. To evaluate \( b \), it is thus necessary to solve the implicit equation \( 2L(r_e) = L_T \). For the Sérsic profile \( (b = b_n) \), as is known, this produces equation (A7), given above. For the core-Sérsic model, \( b \) is a function of the various parameters \((\alpha, \gamma, r_h, \text{and } r_e)\) and can be determined by solving the following relation:

\[
2 \int_{b(r_h/r_e)^{1/n}}^{b(r_e/r_e)^{1/n}} e^{-x} x^{n(\gamma+\alpha)} \left[ x^{n\alpha} - (b^n r_h/r_e)^\alpha \right]^{(2\gamma-\alpha)/\alpha} dx = \Gamma(2n) + \gamma \left[ 2n, b(r_h/r_e)^{1/n} \right] - 2\gamma(2n, b) \quad \text{(A10)}
\]

This assumes that \( \alpha > 0 \). As \( r_h \to 0 \), we recover the Sérsic expression. In the particular case \( \alpha \to \infty \) (sharp transition between inner power-law and outer Sérsic regimes), the equation simplifies to

\[
\left[ 1/(2 - \gamma) \right] (r_h/r_e)^2 = (n/b^2) e^{b(r_h/r_e)^{1/n}} \left\{ \Gamma(2n) + \gamma \left[ 2n, b(r_h/r_e)^{1/n} \right] - 2\gamma(2n, b) \right\} \quad \text{(A11)}
\]

In practice, as long as \( r_h \ll r_e \) and \( \gamma < 1 \), the above equation can be simplified even more:

\[
\Gamma(2n) + \gamma \left[ 2n, b(r_h/r_e)^{1/n} \right] \approx 2\gamma(2n, b) \quad \text{(A12)}
\]

**A3. Local Logarithmic Slope \( \gamma' \)**

Rest et al. (2001) introduced \( \gamma' \) as a measure of the (logarithmic) gradient of the luminosity profile at some specific radius \( r' \):

\[
\gamma' = -\left( \frac{d \log I}{d \log r} \right)_{r'} \quad \text{(A13)}
\]

For the Nuker law, \( \gamma' \) is (e.g., Rest et al. 2001, eq. [8]):

\[
\gamma' = \frac{\gamma + \beta (r'/r_h)\alpha}{1 + (r'/r_h)^\alpha} \quad \text{(A14)}
\]
As Rest et al. noted, this is a more accurate description of the local logarithmic slope than the Nuker-law parameter $\gamma$ when the transition between the two power-law regimes is soft (i.e., small $\alpha$). For the Sérsic profile we have

$$\gamma' = b/n(r'/r_e)^{1/n}. \quad (A15)$$

Finally, for the core-Sérsic model:

$$\gamma' = b/n(1/r_e)^{1/n} \rho^{1/n} (r^{1/n} + r_h^n)^{(1/n)-1} + \gamma(r_h/r')^\alpha \left( 1 + (r_h/r')^\alpha \right)^{-1}. \quad (A16)$$

As $r_h \to 0$, we recover the Sérsic expression. As $\alpha \to \infty$, $\gamma'$ is described by the Sérsic value outside $r_h$ and is equal to $\gamma$ inside.

### A4. TOTAL LUMINOSITY

We assume the object is circular. If the galaxy is elliptical the following expressions must be multiplied by $b/a$, where $a$ and $b$ are semimajor and semiminor axes, respectively. The total luminosity is defined as

$$L_T = \int_0^{2\pi} \int_0^{+\infty} I(r) r \, dr \, d\theta. \quad (A17)$$

For a Sérsic profile the total luminosity is then

$$L_T = \frac{2\pi n}{b^{2n}} \Gamma(2n) I(0) r_e^2, \quad (A18)$$

while for the core-Sérsic model it is

$$L_T = 2\pi I n(r_e/b^n)^2 \int_{b(r_h/r_e)^{1/n}}^{+\infty} e^{-x} x^{(\gamma+\alpha)-1} \left[ x^{\alpha n} - (b^n r_h/r_e)^{\alpha n} \right]^{2(1-\gamma)/\alpha} \, dx. \quad (A19)$$

This expression is valid for $\alpha > 0$. As $r_h \to 0$, we recover the Sérsic expression. In the particular case $\alpha \to \infty$, this expression becomes:

$$L_T = 2\pi I \left[ \frac{r_h^2}{(2 - \gamma)} \right] + e^{b(r_h/r_e)^{1/n}} n(r_e/b^{2n}) \left\{ \Gamma(2n) - \gamma \left[ 2n, b(r_h/r_e)^{1/n} \right] \right\}. \quad (A20)$$

### APPENDIX B

#### CONTOUR MAPS AND ELLIPSE FITS

In Figure 10 we display the isophotal contour maps and ellipse fits for the WFPC2 mosaics of each of the galaxies we analyzed. Details of the data reduction can be found in § 3.2.

### APPENDIX C

#### GALAXIES REJECTED AS PROBABLE S0’s

The following galaxies met our selection criteria for size and for the existence of WFPC2 archival images in the appropriate filters, but were judged to have significant disks and thus be possible S0 galaxies, despite their formal classification as ellipticals. We err on the conservative side by considering the presence of bars and rings to be evidence for an S0 galaxy; evidence for a bar includes the appearance of the isophotes, peaks in ellipticity and accompanying position-angle twists in the ellipse fits, and typical bar appearance in unsharp masks (see, e.g., Erwin & Sparke 2003). We also use evidence from our attempts to fit the extranuclear ($r > 1''$) light profiles (derived from the mosaic images) with both pure Sérsic and disk+bulge models: i.e., there are some galaxies for which Sérsic+exponential is clearly a better fit than pure Sérsic.

**NGC 596** (source Lauer et al. 1995): Nieto et al. (1992) argued that this was actually an SB0 galaxy; Faber et al. (1997) also note that this galaxy has “an S0-like outer envelope.” Our fits to the light profile also suggest a disk+bulge morphology.

**NGC 2592** (source: Rest et al. 2001): Kinematic evidence from Rix, Carollo, & Freeman (1999) strongly suggests this is an S0 galaxy; in addition, there is clear evidence of a bar in the PC isophotes and unsharp masks.

**NGC 2699** (source: Rest et al. 2001): Kinematic evidence from Rix et al. (1999) strongly suggests this is an S0 galaxy; in addition, there is clear evidence of a bar in the PC image (Rest et al. 2001 pointed to this galaxy as a providing a good example of a misaligned inner structure, e.g., a bar).
Fig. 10.—Isophotes and ellipse fits for WFPC2 mosaic images of the 21 elliptical galaxies in our sample. Contour plots of the isophotes show the entire WFPC2 array; the coordinate axes are centered on the galaxy nucleus. Isophotes are logarithmically scaled and have been smoothed with a 5 pixel wide median filter prior to contouring.
Fig. 10.—Continued
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NGC 2778 (source: Rest et al. 2001): Kinematic evidence from Rix et al. (1999) strongly suggests this is an S0 galaxy; in addition, there is good evidence for a bar in the PC image. Analysis of the light profile in Kent (1985) and Erwin, Caon, & Graham (2004a) also supports an S0 (i.e., bulge+outer disk) interpretation.

NGC 3608 (source: Lauer et al. 1995): The light profile is significantly better fitted with a disk+bulge model than by a pure Sérsic model; see Erwin et al. (2004a).

NGC 4121 (source: Rest et al. 2001): There is clear evidence for a bar in the PC image (“misaligned inner structure”) in Rest et al., and the extranuclear light profile is much better fitted with a composite (bulge+disk) model than by a single Sérsic component.

NGC 4564 (source: Rest et al. 2001): Unsharp masking of the PC image indicates that the elliptical feature dominating the isophotes is a stellar ring, which we judge to be a signature of a significant disk; there is some evidence for a nuclear bar as well. Analysis of the light profile in Erwin et al. (2004a) also supports an S0 (i.e., bulge+outer disk) interpretation.

NGC 4648 (source: Rest et al. 2001): A very clear, strong bar dominates the inner isophotes of the PC image (“misaligned inner structure” in Rest et al.).

NGC 5812 (source: Rest et al. 2001): The light profile is somewhat better fitted with a disk+bulge model than by a pure Sérsic model; there is also weak evidence for a possible bar or ring in the r ≈ 2–5″ isophotes. This is probably the most uncertain “S0” classification in our rejected set.

NGC 5813 (source: Rest et al. 2001): The ellipticity steadily increases outward in this galaxy, from ~0.1 near the center to ~0.3 at large radii, which is possible evidence for an outer disk. Analysis of the light profile indicates a disk+bulge structure as well.

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