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Finite temperature effects on the collapse of trapped Bose-Fermi mixtures

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By using the self-consistent Hartree-Fock-Bogoliubov-Popov theory, we present a detailed study of the mean-field stability of spherically trapped Bose-Fermi mixtures at finite temperature. We find that, by increasing the temperature, the critical particle number of bosons (or fermions) and the critical attractive Bose-Fermi scattering length increase, leading to a significant stabilization of the mixture.

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and the modified Bogoliubov-deGennes (BdG) equations
\[ \mathcal{H}_b = \psi^+(\mathbf{r}) \left[ -\frac{\nabla^2}{2m_b} + V_{\text{ext}}(\mathbf{r}) - \mu_b \right] \psi(\mathbf{r}) + \frac{g_{bb}}{2} \psi^+ \psi^+ \psi^+ \psi, \]
\[ \mathcal{H}_f = \phi^+(\mathbf{r}) \left[ -\frac{\nabla^2}{2m_f} + V_{\text{ext}}(\mathbf{r}) - \mu_f \right] \phi(\mathbf{r}), \]
\[ \mathcal{H}_{bf} = g_{bf} \psi^+(\mathbf{r}) \psi(\mathbf{r}) \phi^+(\mathbf{r}) \phi(\mathbf{r}), \]
where \( \psi(\mathbf{r}) \) (\( \phi(\mathbf{r}) \)) is the Bose (Fermi) field operator that annihilates an atom at position \( \mathbf{r} \). The first and second terms in the brackets contain the kinetic-energy operators and the external trapping potentials \( V_{\text{ext}}(\mathbf{r}) = m_b \omega_b^2 r^2/2 \) and \( V_{\text{ext}}(\mathbf{r}) = m_f \omega_f^2 r^2/2 \) for the bosonic and the fermionic species with masses \( m_b \) and \( m_f \), respectively. In the dilute regime, we have considered two types of contact interactions: the interactions between bosons, and the interactions between bosons and fermions. They are parametrized by the coupling constants \( g_{bb} = 4\pi \hbar^2 a_{bb}/m_b \) and \( g_{bf} = 2\pi \hbar^2 a_{bf}/m_r \), respectively, in terms of the s-wave scattering length \( a_{bb} \) and \( a_{bf} \), with \( m_r = m_b m_f/(m_b + m_f) \) being the reduced mass. We have neglected here the fermion-fermion interactions since for spin-polarized fermions the s-wave contact interaction is prohibited by the Pauli principle. The next leading order, p-wave interaction is small at low energy [30] and will not be considered in the following.

We consider the many-body ground state \( |\Psi\rangle \) of Hamiltonian (1) as a direct product of a symmetric \( N_b \)-body state \( |\Psi_b\rangle \) for the bosonic species and an antisymmetric \( N_f \)-body state \( |\Psi_f\rangle \) for the fermions. With this choice we are not considering the possible correlation effects beyond the mean-field approximation [31]. The density Hamiltonian describing the Bose-Fermi coupling can therefore be decoupled in a self-consistent mean-field manner, namely,

\[ \mathcal{H}_{bf} \simeq g_{bf} \left[ \psi^+ \psi \langle \phi^+ \phi \rangle + \langle \psi^+ \psi \rangle \phi^+ \phi - \langle \psi^+ \psi \rangle \langle \phi^+ \phi \rangle \right]. \]  

As a result, the bosonic and fermionic atoms experience, respectively, the effective potentials \( V_{\text{eff}}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + g_{bf} n_f(\mathbf{r}) \) and \( V_{\text{eff}}(\mathbf{r}) = V_{\text{ext}}^b(\mathbf{r}) + g_{bf} n_b(\mathbf{r}) \), with \( n_b(\mathbf{r}) \) and \( n_f(\mathbf{r}) \) being the bosonic and fermionic density distributions, respectively. It is then straightforward to apply the HFB-Popov theory for the bosonic species, following Ref. 32. After invoking the Bose symmetry breaking in the equation of motion for the Bose field operator \( \psi(\mathbf{r}) \), we obtain, respectively, the modified Gross-Pitaevskii (GP) equation for the condensate wave function \( \Phi(\mathbf{r}) \),

\[ \mathcal{L}_{GP} \Phi(\mathbf{r}) = 0, \]

and the modified Bogoliubov-deGennes (BdG) equations for the thermal quasiparticle amplitudes \( u_i(\mathbf{r}) \) and \( v_i(\mathbf{r}) \),

\[ \mathcal{L}_{GP} + g_{bb} n_b(\mathbf{r}) \] \[ u_i(\mathbf{r}) + g_{bb} n_c(\mathbf{r}) v_i(\mathbf{r}) = \epsilon_i u_i(\mathbf{r}), \]
\[ \mathcal{L}_{GP} + g_{bb} n_c(\mathbf{r}) \] \[ v_i(\mathbf{r}) + g_{bb} n_b(\mathbf{r}) u_i(\mathbf{r}) = -\epsilon_i v_i(\mathbf{r}), \]

where the operator

\[ \mathcal{L}_{GP} = -\frac{\nabla^2}{2m_b} + V_{\text{ext}}^b - \mu_b + g_{bb} (n_c(\mathbf{r}) + 2n_T(\mathbf{r})) + g_{bf} n_f(\mathbf{r}), \]

is the generalized GP Hamiltonian that includes the mean-field contributions generated by the interaction with the thermal cloud and the fermionic species. Once these wave functions have been determined, the local density of the condensate and of the noncondensate, and the total bosonic density distribution can be calculated according to

\[ n_c(\mathbf{r}) = |\Phi(\mathbf{r})|^2, \]
\[ n_T(\mathbf{r}) = \sum_i \left( \frac{|u_i(\mathbf{r})|^2 + |v_i(\mathbf{r})|^2}{e^{\beta \epsilon_i} - 1} + |v_i(\mathbf{r})|^2 \right), \]
\[ n_b(\mathbf{r}) = n_c(\mathbf{r}) + n_T(\mathbf{r}), \]

with \( \beta = 1/k_B T \) being the inverse temperature. To solve the modified GP and BdG equations, one has to evaluate the fermionic density distribution \( n_f(\mathbf{r}) \). To this aim, we employ the finite-temperature Thomas-Fermi approximation (TFA) [34].

\[ n_f(\mathbf{r}) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta \epsilon_p + V_{\text{ext}}^f + g_{bf} n_b(\mathbf{r})} + 1} = m_f \sqrt{2\pi^2} \int_{-\infty}^{+\infty} de e^{\beta(\epsilon - \mu_f)} + 1 \left( \epsilon - V_{\text{ext}}^f - g_{bf} n_b \right)^{1/2}. \]

Specially, for the range of \( N_f \) considered here \( (N_f \simeq 10^4) \), it was shown that TFA is a good approximation for a mixture with repulsive Bose-Fermi interaction at zero temperature [35]. In our case, this approximation is expected to work even better, since the quantum fluctuations are suppressed by the finite temperature and the increased density due to the Bose-Fermi attraction.

Equations (4)–(7) form a closed set of equations that we have referred to as the “HFB-Popov” equations for a dilute Bose-Fermi mixture. The simultaneous solution of this set of equations gives the temperature-dependent density profiles of the condensate, of the noncondensate, and of the degenerate Fermi gas, from which we can extract the stability conditions of the system. We have numerically solved these coupled equations by an iterative scheme, as described in Ref. 20.

With above tools we investigate the effects of the temperature upon the collapse of a trapped Bose-Fermi mixture induced by the attractive Bose-Fermi interaction. Rather than taking the realistic experimental conditions, in this paper we present a general analysis of finite temperature effects, and restrict ourselves to spherical symmetric systems. The reason is that for this geometry the temperature-dependent density profiles of the system are easily calculated. On the contrary, for anisotropic systems, though the formalism given in the present paper is...
suitable as well, the numerical calculation is computationally much more involved. In particular, for the highly anisotropic traps used in the experiments at the European Laboratory for Non-linear Spectroscopy (LENS) [4, 21] it is unlikely to obtain useful information without the use of further approximation. In the following, we consider equal masses for the two species $m = m_b = m_f$ and identical trapping frequencies $\omega = \omega_b = \omega_f$. The oscillator length $l = (\hbar/\omega m)^{1/2}$ and trap energy $\hbar\omega$, respectively, serve as fundamental length unit and energy unit. We also take the transition temperature of a non-interacting Bose gas with $N_b = 5 \times 10^4$ atoms, $T_c = 0.94\hbar\omega N_b^{1/3}/k_B = 34.63\hbar\omega/k_B$, as a characteristic temperature. In most calculations, we have set $T = 0.67T_c$ that corresponds to a typical detection limit for the measurement of the temperature.

First we consider the effects of the temperature upon the density profiles of the mixture. The upper and lower rows in Fig. 1 show the density profiles of configurations with $N_b = N_f = 5 \times 10^4$ particles for three different values of the s-wave Bose-Fermi scattering length $a_{bf}/l$ at two temperatures: $T = 0.005T_c$ and $T = 0.67T_c$. The value of $a_{bf}/l = 0.005$ has been fixed, which corresponds to $a_{bf} \approx 100a_{Bbhr}$ for a typical trap with $l = 1\mu m$. Our results at the lower temperature $T = 0.005T_c$ are in good agreement with the findings by Roth and co-workers [23, 24], that is, the mutual attraction between bosons and fermions results in an enhancement of both densities in the overlap region. In particular, as shown in Fig. (1c), both the bosonic and fermionic densities grow substantially when the mixture is close to the instability point (note that at $T = 0.005T_c$ the critical value of $a_{bf}/l$ is $-0.0193$). This enhancement, however, is much reduced at a finite temperature, whose effect is a broadening of the density distributions of the condensate and of the Fermi gas and therefore reduces their center densities. As can be seen in Fig. (1f), at $T = 0.67T_c$ both densities is decreased by a factor of 2 compared to the lower temperature case. As a consequence, the mixture in this case is expected to be much stabilized against collapse.

In a simplified model [4], the collapse or the instability of the mixture is governed by the balance between the kinetic energy of fermions and the mutually attractive mean-field generated by the Bose-Fermi interaction (see, for example, the discussion above the Eq. (11) in Ref. [4]). If the Bose-Fermi attraction becomes too strong, i.e., the numbers of bosons (or fermions) or the scattering length $a_{bf}$ becomes too large, the attractive mean field cannot be stabilized by the kinetic energy anymore, so both density distributions grow indefinitely within the overlap region and collapse. According to the study reported in Refs. [4, 23, 24, 25], the onset of this mean-field instability is monitored by the failure of the convergence during the iterative procedure. In this manner, we can determine the critical s-wave Bose-Fermi scattering length $a_{bf}$ or critical particle numbers $N_b^{crit}$ and $N_f^{crit}$ beyond which the collapse occurs.

In Fig. 2, we show the critical particle number as a function of the s-wave Bose-Fermi scattering length $a_{bf}/l$ with fixed $a_{bb}/l = 0.005$ for two temperatures. In

![FIG. 1: Radial density profiles of a Bose-Fermi mixture with $N_b = N_f = 5 \times 10^4$ for different temperatures and Bose-Fermi interaction strengths. The condensate and noncondensate densities $n_c(r)$ and $n_f(r)$ are given, respectively, by the solid and dash-dotted lines (left scale), and the fermion density $n_f(r)$ by dashed line (right scale). To visualize $n_f(r)$ is enlarged by a factor of ten. The upper and lower rows show, respectively, examples with decreasing Bose-Fermi attraction at $T/T_c = 0.005$ and 0.67, where $T_c = 0.94\hbar\omega N_b^{1/3}/k_B$ is the transition temperature of a non-interacting Bose-Einstein condensate in the thermodynamic limit. We fix the boson-boson scattering length $a_{bb}/l = 0.005$.](image-url)
this calculation we have kept equal numbers of bosons and fermions \(N_b = N_f\). At the lower temperature \(T = 0.005T_c\), we observed that even a small decrease of Bose-Fermi attraction can reduce the critical particle number significantly, i.e., \(N_c^{\text{crit}}\) changes by a factor of 2 when \(a_{bf}/l\) changes by 5 percent. This strong dependence of the criticality on \(a_{bf}/l\) is in accordance with the scaling law for the critical particle number: \(N_c^{\text{crit}} \sim |a_{bf}|^{-\alpha}\) with \(\alpha = 12\), as discussed in Refs. [6, 22, 25]. The presence of a moderate temperature \(T = 0.67T_c\) results in a substantial increase of the critical particle number. For example, at \(a_{bf}/l = -0.022\), \(N_c^{\text{crit}}\) grows from \(1.1 \times 10^4\) to \(3.6 \times 10^4\) with the inclusion of the temperature. Apart from this growth, the critical particle number is still strongly dependent on \(a_{bf}/l\). However, the presence of the finite temperature leads to a re-normalization of the scaling exponent, i.e., \(\alpha \approx 8\).

In Figs. (3a) and (3b), we study the critical particle numbers of bosons and fermions as a function of the temperature. We report, respectively, the prediction on the critical particle numbers of bosons (with fixed \(N_f = 5 \times 10^4\)) and fermions (with fixed \(N_b = 5 \times 10^4\)). For comparison, in Fig. (3a), we also show the critical number of the condensed atoms against temperature (dashed line). As expected, the critical particle number for each species increase with increasing the temperature. The dependence is highly nonlinear. Below \(0.5T_c\), the critical particle number varies slowly with the temperature, whereas above \(0.5T_c\) it rises up steeply. In addition, the critical particle number of fermions grows more rapidly than that of bosons against the temperature.

Finally, we have built a region of stability of the Bose-Fermi mixture in Fig. 4, as a function of the number of...
atoms, for the cases of $a_{bb}/l = 0.005$ and $a_{bf}/l = -0.020,$ for three values of the temperature: $T/T_C = 0.005$, $T/T_C = 0.50$ and $T/T_C = 0.67$. Each of the curves marks the limit of stability. For numbers of bosons or fermions below the stability limit the mixture is stable, otherwise the mixture is unstable against mean-field collapse. Figure 4 indicates that the region of the stability broadens with increasing temperature. This behavior emphasizes again that the inclusion of a temperature gives rise to a significant stabilization of the mixtures.

In conclusion, we have investigated the mean-field stability of a spherically trapped binary Bose-Fermi mixture at finite temperature. We solved the coupled HFB-Popov equations numerically and obtained the critical particle number as a function of the temperature and the $s$-wave Bose-Fermi scattering length. We have shown that the critical particle number and the critical Bose-Fermi scattering length increase with the inclusion of a moderate temperature that corresponds to the typical experimental detection limit. This leads to a significant stabilization of the Bose-Fermi mixtures.

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