The Majorana fermion [1]—a particle that is its own antiparticle—has attracted considerable attention in a wide area of physics [2]. Particular interest comes from its non-Abelian exchange statistics, which is crucial for topological quantum computation [3,4]. Two well-separated Majorana fermions may form a nonlocal fermionic state, as a nonlocal qubit for inherently fault-tolerant quantum memory. As a portal for future quantum technology, the realization of Majorana fermions in a highly controllable manner is of great importance for future quantum technology, the realization of Majorana qubit for inherently fault-tolerant quantum memory. As a portal for future quantum technology, the realization of Majorana fermions in a highly controllable manner is of great importance.

In this Rapid Communication, we examine the possibility of observing Majorana fermions in the vortex core of a spin-orbit-coupled ultracold atomic Fermi gas in 2D harmonic traps. This is a scenario discussed earlier by several researchers [11–13], based on a theoretical concept originated by Jackiw and Rossi, who predicted the existence of vortex-core Majorana fermions in (2 + 1)-dimensional Dirac theory [14]. Here, we perform a fully microscopic calculation with a Bogoliubov–de Gennes (BdG) equation, which enables simulations with realistic experimental parameters. Our study is motivated by the recent creation of non-Abelian gauge fields in a Bose-Einstein condensate (BEC) of $^{87}$Rb atoms [15] and its possible realization in fermionic $^{6}$Li atoms [16].

We find that by increasing a Zeeman field, a topological superfluid emerges from the trap edge and extends gradually to the whole Fermi cloud, supporting zero-energy states (ZESS) at the vortex core and trap edge. This topological phase transition is detectable through a sudden change in the atomic density inside the vortex core, associated with the occupation of the Majorana state. We show that the wave function of the Majorana fermions can be inferred from the local density of states (LDOS) at the core.

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Mean-field BdG equation. We consider a trapped 2D atomic Fermi gas subject to Rashba spin-orbit coupling $V_{SO}(r) = -i\lambda(\partial_{x} + i\partial_{y})$ and a Zeeman field $h$, which may be prepared in a single oblate optical trap $V(r,z) = M[\omega_{r}^{2}r^{2} + \omega_{z}^{2}z^{2}] / 2$ with trapping frequencies $\omega_{r}, \omega_{z}$. We note that 2D Fermi gas has been realized recently in experiments [17, 18]. The system is described by $H = \int d^{2}r[H_{0}(r) + H_{t}(r)]$, where $H_{0}(r) = \sum_{\sigma = \uparrow, \downarrow} \psi_{\sigma}^{\dagger}(r)H_{\sigma}(r)\psi_{\sigma} + [\psi_{\uparrow}^{\dagger}(r)V_{SO}(r)\psi_{\downarrow} + \text{H.c.}]$ (1) and $H_{t}(r) = U_{0}[\psi_{\uparrow}^{\dagger}(r)\psi_{\uparrow}(r)\psi_{\downarrow}(r)\psi_{\downarrow}(r) - \text{H.c.}]$, where $H_{\sigma}(r) = -\hbar^{2}V_{\sigma}(r) + M\omega_{r}^{2}r^{2} / 2 - \mu - \hbar\omega_{z}z$. Here $\mu$ is the single-particle Hamiltonian in reference to the chemical potential $\mu$. The interaction strength $U_{0}$ is to be regularized via $1 / U_{0} + \sum_{k} 1 / (\hbar^{2}k^{2} / 2M + E_{k}) = Mf(\sqrt{\mu}E_{k}/E)$. Here $E_{k}$ is the binding energy of the two-body bound state [19,20] and $E > 0$ is the relative collision energy.

The low-energy fermionic quasiparticles are solved by the mean-field BdG approach, $\mathcal{H}_{BdG}\Psi_{\sigma}(r) = E_{\sigma}\Psi_{\sigma}(r)$. Using the convention for Nambu spinors $\Psi_{\sigma}(r) = [u_{\uparrow\sigma},u_{\downarrow\sigma},v_{\uparrow\sigma},v_{\downarrow\sigma}]^{T}$, the BdG Hamiltonian reads $\mathcal{H}_{BdG} = \begin{bmatrix} H_{\uparrow\uparrow}(r) & V_{SO}(r) & 0 & -\Delta(r) \\ V_{SO}^{\dagger}(r) & H_{\down\down}(r) & \Delta^{*}(r) & 0 \\ 0 & \Delta^{*}(r) & -H_{\down\up}(r) & V_{SO}^{\dagger}(r) \\ -\Delta(r) & 0 & V_{SO}(r) & -H_{\up\up}(r) \end{bmatrix}$ (2), where $\Delta = -(U_{0}/2)\sum_{\sigma}\sum_{\eta}u_{\uparrow\sigma}^{\dagger}v_{\eta\sigma}^{\dagger}f(E_{\eta}) + u_{\down\sigma}^{\dagger}v_{\eta\sigma}f(-E_{\eta})$ is the order parameter, to be solved self-consistently in conjunction with the atomic densities, $n_{\sigma}(r) = (1 / 2)\sum_{\eta} |u_{\sigma\eta}|^{2} f(E_{\eta}) + |v_{\eta\sigma}|^{2} f(-E_{\eta})$. Here $f(x) = 1 / (e^{x/\hbar^{2}T} + 1)$ is the Fermi distribution function. The chemical potential $\mu$ is determined by the total atom number $N = \int d^{2}r n_{\uparrow}(r) + n_{\downarrow}(r)$. With a single vortex at trap center, we take $\Delta(r) = \Delta(r)e^{-i\varphi}$ and decouple the BdG equation into different angular momentum channels indexed by an integer $m$. The quasiparticle wave functions take the form $[u_{\uparrow\sigma}(r)e^{-i\varphi},u_{\down\sigma}(r),v_{\up\sigma}(r)e^{i\varphi},v_{\down\sigma}(r)]e^{im\theta} / \sqrt{2\pi}$. We have...
solved self-consistently the BdG equations by using the basis expansion method. For the results presented here, we have taken $N = 400$ and $T = 0$. We have used $E_0 = 0.2 E_F$ and $\lambda k_F / E_F = 1$, which are typical parameters that can be readily realized in a 2D $^{40}$K Fermi gas [18]. Other sets of parameters, with varying interaction strength, SO coupling strength, and temperature, have also been tried.

The use of Nambu spinor representation leads to an inherent particle-hole redundancy built into the BdG Hamiltonian. $\mathcal{H}_{\text{BdG}}$ is invariant under the particle-hole transformation, $u_\sigma (r) \rightarrow v_\sigma (r)$ and $E_\eta \rightarrow - E_\eta$. Thus, every eigenstate with energy $E$ has a partner at $-E$. These two states describe the same physical degrees of freedom, as the Bogoliubov quasiparticle operators associated with them satisfy $\Gamma_E = \Gamma_{E}^{-1}$. In the expressions for order parameter and atomic density, this redundancy has been removed by multiplying a factor of $1/2$.

Majorana fermions. The particle-hole redundancy, however, is very useful to illustrate a nontrivial feature when the Zeeman field $h$ is beyond a threshold, $h > h_c$, the system is in a topological state [8], hosting ZESs within the energy gap. Due to $E = 0$, the associated quasiparticle operators satisfy $\Gamma_0 = \Gamma_{0}^{-1}$. Thus, a zero-energy quasiparticle is its own antiparticle—exactly the defining feature of a Majorana fermion [2]. Because of the redundant particle-hole representation, the ZES or Majorana fermion is half of an ordinary fermion and thus must always come in pairs. Each of the paired states, localized separately in real space, can be hardly pushed away from $E = 0$ by a local perturbation [4,6], giving rise to the intrinsic topological stability enjoyed by Majorana fermions. It is straightforward to show from the BdG Hamiltonian that the wave function of Majorana fermions should satisfy either $u_\sigma (r) = v_\sigma (r)$ or $u_\sigma (r) = -v_\sigma^* (r)$. The former follows directly from the particle-hole symmetry. For the latter, the related quasiparticle operator satisfies $\tilde{\Gamma}_0 = -\Gamma_{0}^\dagger$. This is necessary for expressing an ordinary fermion using paired Majorana fermions [21].

Phase diagram. Figure 1 reports the phase diagram [Fig. 1(a)] along with the quasiparticle energy spectrum of different phases [Figs. 1(d)–1(f)] in the presence of a single vortex. By increasing the Zeeman field, the system evolves from a nontopological state (NS) to a topological state (TS) through an intermediate mixed phase in which NS and TS coexist. The mixed phase, unique for a trapped system, can be easily understood from the point of view of a local density approximation, in which the local chemical potential $\mu(r) = \mu - M_0 r^2 / 2$ and the order parameter $\Delta(r)$ decrease continuously away from the trap center. As a result, the local critical Zeeman field $h_c(r) = \sqrt{\mu^2 (r) + \Delta^2 (r)}$ becomes smaller at the trap edge, as shown in [Fig. 1(b)] for $h = 0.5 E_F$. This creates a ring of TS at the outer region where $h > h_c(r)$, surrounding the inner region, which is nontopological. The full topological transition occurs when the Zeeman field is larger than the critical field at the trap center, $h > \sqrt{\mu^2 + \Delta^2 (0)}$, where $\Delta(0)$ is the gap at trap center in the absence of the vortex.

The topological phase transition into TS is well characterized by the low-lying quasiparticle spectrum, which has the particle-hole symmetry $E_{m+1} = -E_{-(m+1)}$. As shown in
The eigenstates with nearly zero energy at $m = -1$, i.e., the two edge states in the mixed phase as well as the outer edge state and CdGM state seems to be more difficult, given rise to an exponentially small splitting (i.e., $E_{\text{ZES}} \approx 5 \times 10^{-6} E_F$). In contrast, the tunneling between the two Majorana states, which are localized respectively at the vortex core and trap edge, leading to a steady increase of the lowest eigenenergy. We note that the exponentially small energy of Majorana fermions in the full TS, inherent to the finiteness of the trapped Fermi cloud, should be suppressed by increasing the number of total atoms.

Probing Majorana fermions. In the TS, the occupation of the Majorana vortex-core state affects significantly the atomic density and LDOS of the Fermi cloud near the trap center, which in turn gives an unambiguous experimental signature for observing Majorana fermions.

Figure 3 presents the spin-up and spin-down densities at the trap center, $n_\uparrow(0)$ and $n_\downarrow(0)$, as a function of the Zeeman field. In general, $n_\uparrow(0)$ and $n_\downarrow(0)$ increase and decrease, respectively, with increasing field. However, we find a sharp increase of $n_\uparrow(0)$ when the system evolves from the mixed phase to the full TS. Accordingly, a change in slope or kink appears in $n_\downarrow(0)$. The increase of $n_\uparrow(0)$ is associated with the gradual formation of the Majorana vortex-core mode, whose occupation contributes notably to atomic density due to the large amplitude of its localized wave function. In the inset of Fig. 3(b) we plot $n_\downarrow(0)$ at $h = 0.6 E_F$, with or without the contribution of the Majorana mode, which is highlighted by the shaded area. This contribution is apparently absent in the NS. Thus, a sharp increase of $n_\downarrow(0)$, detectable in in situ absorption imaging, signals the topological phase transition and the appearance of the Majorana vortex-core mode. This
analog of the widely used scanning tunneling microscope in a
feature persists at a typical experimental temperature, i.e.,
\( T = 0.1 T_F \). We note that experimentally it is more favorable
to take a time-of-flight imaging of the cloud after an expansion
time, in order to have enough resolution to visualize the vortex
limit across Feshbach resonances. In this way, the vortex core
can be imaged clearly after the time of flight, just as in an
atomic BEC.

In the full TS, the wave function of the Majorana mode
can be determined by measuring the LDOS through spatially
resolved rf spectroscopy [24,25], which provides a cold-atom
analog of the widely used scanning tunneling microscope in a
solid state. We show in Fig. 4 the spin-up and spin-down LDOS
at \( h = 0.6 E_F \), defined as \( \rho_\sigma(r, E) = \langle 1/2 \rangle \sum_\eta |\langle u_\sigma\eta | \delta(E - E_\eta) + |v_\eta| \delta(E + E_\eta) \rangle \), where the \( \delta \) function can be sim-
ulated by a Lorentzian distribution with a suitable energy broadening \( \Gamma \). Inside the vortex core, the contribution from the
Majorana mode and other CdGM states is clearly visible within
the superfluid gap. In the case of \( \Gamma \), \( k_B T < \Delta E \), where \( \Delta E \sim \Delta^2(0)/(2E_F) \) is the energy spacing of CdGM states [22], which
for typical parameters as used in our calculation turns out to be
about 10 nK, the Majorana fermion contribution \( \rho_\sigma(\rho, 0) \) may be singled out. As \( \rho_\sigma(\rho, 0) \propto |\langle u_\sigma | \delta^2 = |v_\eta| \rangle| \),
the spatially resolved rf spectroscopy maps out directly the
wave function of the Majorana vortex-core state.

In closing, we note that 2D ultracold atomic Fermi
gases are an ideal platform for probing and manipulating
Majorana fermions because of unprecedented controllability
and flexibility. This is particularly useful for the purpose of
topological quantum computation, using Majorana fermions
as qubits [4]. For instance, two 2D atomic Fermi gases formed
by a double-well potential along the \( z \) axis, each of which
has a single vortex at the center, can host four Majorana
fermions for carrying out the basic information process. In
this configuration, interwell quantum tunneling of Majorana
fermions is possible, providing another potential means to
detect the interesting topological phase transition.

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[21] The ordinary fermion operator at \( E = 0 \) is given by \( c = \Gamma_0 + \Gamma_0 \). By defining Majorana operators \( \gamma_1 = \Gamma_0 \) and \( \gamma_2 = i \Gamma_0 \), we express \( c = \gamma_1 - i \gamma_2 \), as anticipated.