Abstract—This paper provides linear analysis for a practical ultrasound imaging system with single-element transducers. A proper eighth-order linear ARMA model with inputs and outputs of voltage traces is given to present the transfer characteristics of such a practical system, so that echo signals containing tissue information can be collected and analyzed more properly.

I. INTRODUCTION

Ultrasound is one of the most commonly used modalities in medical imaging nowadays. Its safety, low cost, noninvasiveness, portability, real-timeness and many other advantages make it a valuable tool to image the fine details of soft tissues in medicine. However, its image quality is relatively poorer than those of CT and MRI due to 1) geometric distortion, 2) wave front aberration and 3) multiple reflections in [1].

In order to analyze the ultrasound echo signal, then move the distortions, and hence restore corrupted ultrasound signals, ultrasound signal modeling is of most importance. A number of works have been carried out in this area. J. Arendt Jensen [2] obtained a time-domain analytic expression for the back-scattered radio-frequency signals. Zemp et al. [3] extended the current models to more realistic shift-variant imaging systems and nonstationary random scattering media. Ng et al. [4] proved that for some special case, the point-spread function of the imaging systems is shift-variant only in the axial direction.

After careful analysis of the models studied in [2–4], it is found that they emphasize more on theoretical description than practical implementation. For a typical practical ultrasound imaging system, shown as in Fig.1, the original inputs are ultrasound pulses with voltages usually as high as several hundred volts, while the final outputs are the received echo voltage traces of tens to hundreds of millivolts. All the models studied in [2–4] have employed the same input: particle velocity \( \rho(t) \) in the surface of transducer. In [2, 5], the simulated systems were excited by a pulsed sinusoidal velocity. In practical measurements, however, such particle velocities for different transducers with different input pulses are hardly possible to acquire, let alone the particle velocities for different transducers with different sinusoidal velocity. In practical measurements, however, such circuits and our current experimental instruments, it is proposed in this paper that, transducers can be modeled as a linear system with inputs and outputs of voltage traces. System identification methods can be applied to achieve the transfer characteristics of the imaging system.

The paper is organized as follows. Section II compares our processing procedure with those in [2, 5] and describes the idea on why and how to identify such system. Section III presents our experiments, designs a repetitive test plan and shows the results and discussion. In Section IV, the conclusion and future work on the identified system model are given.

II. PROBLEM DESCRIPTION

In this section, four aspects of problem are addressed.

A. Typical Ultrasound Model and its problem

Typical ultrasound models, shown in Fig.2a in [1, 2, 5] are corresponding to real ultrasound propagation process in Fig.1.

As indicated in Fig.2a, in general, the output voltage trace of ultrasound echo signals is finally obtained after five steps:

1) \( G_T \) relates the transmitted excitation voltage \( u(t) \) to the particle velocity \( \rho(t) \) of the radiating surface;

2) \( H_T \) relates the velocity of radiator to the harmonic incident pressure \( P_i(x, t) \) at an arbitrary point;

3) \( S \) is a measure of scattering strength of the target, thus its output: backscattered acoustical pressure \( P_r(x, t) \) contains information of target tissue;

4) \( H_R \) inverts the received pressure to the harmonic force \( f(t) \) which acts on the transducer;

5) \( G_R \) relates the force to the final output voltage trace \( y(t) \) which accounts for the transducer impulse response and target tissue information as well.

In [1, 2, 5] the excitation applied on model is \( \rho(t) \): the particle velocity on radiation surface of transducers, which,
model is all concerning transducer properties except effects however, is only an intermediary product resulting from pulse excitation \(u(t)\). In fact, it is quite difficult to acquire the velocity measurements when testing the validity of such models in practical ultrasound imaging systems.

B. Simplified Ultrasound Signal Model

As a result, it is natural to propose another model with input of pulsed voltage trace \(u(t)\). The five blocks of the model in Fig.2a is first reorganized into three blocks in Fig.2b. The voltage output can be represented as:

\[
y(t) = F_{TR}SF_{RE}u(t),
\]

where \(F_{TR}\), \(S\) and \(F_{RE}\) are operators of transmission process, tissue effects and reception process, respectively.

Then in Fig.2c, \(I\) is defined as perfect reflection operator which reflects the emitted pulse completely back to the transducer. Thus, the output of system in Fig.2c is:

\[
m(t) = F_{TR}IF_{RE}u(t) = F_{TR}F_{RE}u(t) \triangleq F_{TR\&RE}u(t)
\]

As indicated in Fig.2d, the final output \(y(t)\) is written as:

\[
y(t) = F_{TR\&RE}Su(t) = S[m(t)].
\]

Based on existing instruments, \(u(t)\), \(m(t)\) and \(y(t)\) can be measured. Therefore, if the transfer characteristics from \(u(t)\) to \(m(t)\) are estimated successfully, given \(y(t)\), the information of target tissue \(S\) can be easily extracted, which is exactly the ultimate goal of ultrasound imaging.

C. Transducer Modeling

As demonstrated in Sec.II-B, the first step before tissue info estimation is to identify the transfer function of the new model with perfect reflection in Fig.2c. Here since the model is all concerning transducer properties except effects of propagation media, modeling of transducers is to be analyzed first.

In [1, 5], a typical piezoelectric ultrasonic transducer consists of a piezoceramic disk and several mechanical matching layers. The piezoceramic part in Fig.3(a1) can be described by a three-port distributed parameter Mason model like Fig.3(a2). After equivalent transformation, it becomes the simpler second-order organization like Fig.3(a3) consisting of linear components, where \(E\) represents either the input voltage when transmitting or the output voltage when receiving; \(V_1\) and \(V_2\) are particle velocities in the two faces of piezoceramic plate; \(P_1\) and \(P_2\) are mechanical pressure on the two sides; The ideal transformer converts mechanical to electrical variables, changing particle velocity into electric current and pressure into electric voltage, and vice versa; \(Z_i, i = 1,...,7\) are equivalent resistors with certain impedance; \(C_i, i = 1,...,5\) are equivalent capacitors; \(L_i, i = 1,2\) are equivalent inductors.

The mechanical matching layers in Fig.3(b1) can also be represented by distributed parameter second-order models like a two-port model in Fig.3(b2), where \(V_1, V_2, P_1\) and \(P_2\) represents all the same as above notations for any matching layer.

Given linear components in Fig.3, two parts in Fig.3(a3) and Fig.3(b2) can be combined to a whole circuit and regarded as a linear model.
D. System Identification

Given the familiar linear circuit model, it is not unusual to realize it by means of classical Autoregressive Moving-Average models (ARMA). In [6], such ARMA model output vector is expressed as a linear combination of past outputs, $m(t)$, past inputs, $u(t)$ and Gaussian noise input $e(t)$.

1) Model Format:

$$
\sum_{j=0}^{n_a} A_j q^{-j} m(t) = \sum_{j=0}^{n_b} B_j q^{-j} u(t-n_k) + \sum_{j=0}^{n_c} C_j q^{-j} e(t), \quad (4)
$$

where $n_k$ is a time delay between inputs and outputs; $q$ is the delay operator; Hence, in Sec.III, modeling like (4) will be implemented to find out relation between input voltage trace $u(t)$ and output $m(t)$.

2) Model Order: As for the order of models, it is calculated that, orders in Fig.3(a3) and (b2) are all second-order. While including transmission and reception, the whole model can be estimated by eighth-order ARMA models.

III. EXPERIMENTS AND RESULTS

Experiments are designed to be like that in Fig.4. The transducer in use is A382S, 3.5MHz, GE Panametrics.

Four pairs of inputs in Fig.5 and outputs in Fig.6 are applied to estimate the transfer characteristic of the transducer system, in which one pair is employed to construct the ARMA model, and the other three are for model validation.

With the first pair of input-output, the estimated ARMA model is

$$
A(q)m(t) = B(q)u(t-n_k) + C(q)e(t), \quad (5)
$$

where

$$
A(q) = 1 - 0.9022q^{-1} - 0.7663q^{-2} + 0.4359q^{-3} + 0.1934q^{-4} + 0.06403q^{-5} + 0.00128q^{-6} - 0.05434q^{-7} + 0.05081q^{-8};
$$

$$
B(q) = 2.18e^{-5} - 4.288e^{-5}q^{-1} + 7.303e^{-5}q^{-2} + 1.5e^{-5}q^{-3} - 9.57e^{-5}q^{-4} + 8.411e^{-5}q^{-5} + 1.249e^{-5}q^{-6} - 0.0001235q^{-7};
$$

$$
C(q) = 1 + 0.3021q^{-1} - 0.1821q^{-2};
$$

$$
n_k = 250. \quad (9)
$$

Frequency response is shown in Fig.7. The resonant frequency is $3.5MHz$, i.e. $2.2 \times 10^7 rad/s$, which approximately accords with that in Fig.7.

Comparison between model output and real output in input-output pair one is shown in Fig.8. With the estimated model, the simulated outputs with inputs of the other three are present in Fig.9, Fig.10 and Fig. 11, respectively. The difference between the four figures are not quite significant due to the fact that four inputs are quite similar in shape and thus outputs quite alike too, although there are still noticeable differences especially in matching the peaks.

As seen from Fig.8-11, the model of (5), (6), (8) can match the real outputs well enough to be a proper description of such an ultrasound transducer system.

IV. CONCLUSION AND FUTURE WORK

In this paper, an eighth-order linear ARMA model is proposed to represent the transfer characteristics of a practical
ultrasound imaging system with a single-element transducer due to the linear structures of equivalent circuit of transducers. The estimated linear model is shown to perform properly by some practical experiments. With this model, ultrasound echo signal properties without tissue information are analyzed such that, in the future, tissue information can be easily extracted.

However, our modeling still leaves room to be improved. For one thing, only one transducer is used in the experiment. With tests of more transducers, the model can be justified more properly. For another, the input pulses don't vary much from one to another due to the limitation of pulse generation type in the pulser/receiver. It is believed that, given various input signals, the estimated ARMA model would be more accurate. In addition, the effects of propagation media, say water, is not taken into consideration in the modeling.

**REFERENCES**


