Achieving the Best Trade-Off between Computation and Storage in the Cloud

– Cost Model, Benchmarking and Strategies for Datasets Storage of Scientific Applications

by

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To my parents and my wife
Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma, except where due reference is made in the text of the thesis. To the best of my knowledge, this thesis contains no material previously published or written by another person except where due reference is made in the text of the thesis.

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Abstract

Nowadays, scientific research increasingly relies on IT technologies, where large-scale and high performance computing systems (e.g. clusters, grids and supercomputers) are utilised by the communities of researchers to carry out their applications. Scientific applications are usually computation and data intensive, where complex computation tasks take a long time for execution and the generated datasets are often terabytes or petabytes in size. Storing valuable generated application datasets can save their regeneration cost when they are reused, not to mention the waiting time caused by regeneration. However, the large size of the scientific datasets is a big challenge for their storage.

In recent years, cloud computing is emerging as the latest distributed computing paradigm which provides redundant, inexpensive and scalable resources on demand to system requirements. It offers researchers a new way for deploying computation and data intensive applications (e.g. scientific applications) without any infrastructure investments. Large generated application datasets can be flexibly stored or deleted (re-generate whenever needed) in the cloud, since theoretically unlimited storage and computation resources can be obtained from commercial cloud service providers.

With the pay-as-you-go model, the total application cost for generated datasets in the cloud highly depends on the strategy of storing them, e.g. storing all the generated application datasets in the cloud may result in a high storage cost since some datasets may be seldom used but large in size; in contrast, if we delete all the generated datasets and regenerate them every time when needed, the computation cost may be very high too. Hence there is a trade-off between computation and storage in the cloud. In order to reduce the overall application cost, a good strategy is
to find a balance to selectively store some popular datasets and regenerate the rest when needed. This thesis focuses on cost-effective datasets storage of scientific applications in the cloud, which is a leading-edge and challenging topic nowadays. By investigating the niche issue of computation and storage trade-off, we 1) propose a new cost model for datasets storage in the cloud; 2) develop novel benchmarking approaches to find the minimum cost of storing the application data; 3) design innovative runtime storage strategies to store the application data in the cloud.

We start with introducing a motivating example from astrophysics and analyses the problems of computation and storage trade-off in the cloud. Based on the requirements identified, we propose a novel concept of Data Dependency Graph (DDG) and propose an effective datasets storage cost model in the cloud. DDG is based on data provenance, which records the generation relationship of all the datasets. With DDG, we know how to effectively regenerate datasets in the cloud and can further calculate their generation costs. The total application cost for the generated datasets includes both their generation cost and storage cost.

Based on the cost model, we develop novel algorithms which can calculate the minimum cost for storing datasets in the cloud, i.e. the best trade-off between computation and storage. This minimum cost is a benchmark for evaluating the cost-effectiveness of different storage strategies in the cloud. For different situations, we develop different benchmarking approaches with polynomial time complexity for a seemingly NP-hard problem, where 1) the static on-demand approach is for the situation that only occasional benchmarking is requested; 2) the dynamic on-the-fly approach is suitable for the situation that more frequent benchmarking is requested at runtime.

We develop novel cost-effective storage strategies for users to facilitate at runtime of the cloud. Different from the minimum cost benchmarking approach, sometimes users may have certain preferences on storing some particular datasets due to various reasons rather than cost, e.g. guaranteeing immediate access to certain datasets. Hence, users’ preferences should also be considered in a storage strategy. Based on these considerations, we develop two cost-effective storage strategies for different situations: 1) the cost rate based strategy is highly efficient with fairly
reasonable cost-effectiveness, and 2) the local-optimisation based strategy is highly cost-effective with very reasonable time complexity.

To the best of our knowledge, this thesis is the first comprehensive and systematic work investigating the issue of computation and storage trade-off in the cloud in order to reduce the overall application cost. By proposing innovative concepts, theorems and algorithms, the major contribution of this thesis is that it helps bring the cost down dramatically for both cloud users and service providers to run computation and data intensive scientific applications in the cloud.
The Author’s Publications

Books:


Book Chapters:


Journals:


**Conferences:**


**Journal submissions (under review):**

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Chapter 1

Introduction

This thesis investigates the trade-off between computation and storage in the cloud. This is a brand new and significant issue for deploying applications with the pay-as-you-go model in the cloud, especially computation and data intensive scientific applications. The novel research reported in this thesis is for both cloud service providers and users to reduce the cost of storing large generated application datasets in the cloud. A suite consisting of a novel cost model, benchmarking approaches and storage strategies is designed and developed with the supports of new concepts, solid theorems and innovative algorithms. Experimental evaluation and case study demonstrate that our work helps to bring the cost down dramatically for running the computation and data intensive scientific applications in the cloud.

This chapter introduces the background and key issues of this research. It is organised as follows. Section 1.1 gives a brief introduction to running scientific applications in the cloud. Section 1.2 outlines the key issues of this research. Finally, Section 1.3 presents an overview for the remainder of this thesis.

1.1 Scientific Applications in the Cloud

Running scientific applications usually needs not only high performance computing resources but also massive storage [34]. In many scientific research fields, like astronomy [33], high-energy physics [61] and bio-informatics [65], scientists need to analyse a large amount of data either from existing data resources or collected from
physical devices. During these processes, a large amounts of new data might also be generated as intermediate or final products [34]. Scientific applications are usually data intensive [36, 61], where the generated datasets are often terabytes or even petabytes in size. As reported by Szalay et al. in [74], science is in an exponential world and the amount of scientific data will double every year over the next decade and future. Producing scientific datasets involves a large number of computation intensive tasks, e.g. with scientific workflows [35], hence taking a long time for execution. These generated datasets contain important intermediate or final results of the computation, and need to be stored as valuable resources. This is because: 1) data can be reused - scientists may need to re-analyse the results or apply new analyses on the existing datasets [16]; 2) data can be shared - for collaboration, the computation results may be shared, hence the datasets are used by scientists from different institutions [19]. Storing valuable generated application datasets can save their regeneration cost when they are reused, not to mention the waiting time caused by regeneration. However, the large size of the scientific datasets is a big challenge for their storage. Hence, popular scientific applications are often deployed in grid or HPC (High Performance Computing) systems [61] because they have high performance computing resources and/or massive storage. However, building and maintaining a grid or HPC system is extremely expensive and it cannot be easily made available for scientists all over the world to utilise.

In recent years, cloud computing is emerging as a latest distributed computing paradigm which provides redundant, inexpensive and scalable resources on demand to system requirements [42]. Since late 2007 when the concept of cloud computing was proposed [83], it has been utilised in many areas with certain success [17, 45, 21, 62]. Meanwhile, cloud computing adopts a pay-as-you-go model where users are charged according to the usage of cloud services such as computation, storage and network\(^1\) services like conventional utilities in everyday life (e.g. water, electricity, gas and telephony) [22]. Cloud computing systems offer a new way for deploying computation and data intensive applications. As IaaS (Infrastructure as a Service) is a very popular way to deliver computing resources in the cloud [1], the

\(^1\) In this thesis, we only investigate the trade-off between computation and storage, where network is not incorporated. Please refer to Section 3.2.2 for detailed explanations.
heterogeneity of computing systems [93] of one service provider can be well shielded by virtualisation technology. Hence, users can deploy their applications in unified resources without any infrastructure investment, where excessive processing power and storage can be obtained from commercial cloud service providers. Furthermore, cloud computing systems offer a new paradigm that scientists from all over the world can collaborate and conduct their research jointly. Cloud computing systems are usually based on the Internet, scientists can upload their data and launch their applications in the cloud from anywhere in the world via the Internet. As all the data are managed in the cloud, it is easy to share data among scientists.

However, new challenges also arise when we deploy a scientific application in the cloud. With the pay-as-you-go model, the resources need to be paid for use, hence the total application cost for generated datasets in the cloud highly depends on the strategy of storing them, e.g. storing all the generated application datasets in the cloud may result in a high storage cost since some datasets may be seldom used but large in size; in contrast, if we delete all the generated datasets and regenerate them every time when needed, the computation cost may be very high too. Hence there should be a trade-off between computation and storage for deploying applications, which is an important yet challenging issue in the cloud. By investigating this issue, this research proposes a new cost model, novel benchmarking approaches and innovative storage strategies, which would help both cloud service providers and users to reduce the application cost in the cloud.

1.2 Key Issues of this Research

In the cloud, the application cost highly depends on the strategy of storing the large generated datasets due to the pay-as-you-go model. A good strategy is to find a balance to selectively store some popular datasets and regenerate the rest when needed, i.e. finding a trade-off between computation and storage. However, the generated application datasets in the cloud often have dependencies, i.e. computation task can operate on one or more datasets and generate new one(s). Whether storing or deleting an application dataset impacts on not only the cost of the dataset itself,
but also other datasets in the cloud. To achieve the best trade-off and utilise it to reduce the application cost, we need to investigate the following issues:

1) **Cost model.** Users need a new cost model that can represent the cost that they actually spend on their applications in the cloud. Theoretically, users can get unlimited resources from the commercial cloud service providers for both computation and storage. Hence, for the large generated application datasets, users can flexibly choose how many to store and how many to regenerate. Different storage strategies lead to different consumptions of computation and storage resources and finally lead to different total application costs. The new cost model should be able to represent the cost of the applications in the cloud, which is the trade-off between computation and storage.

2) **Minimum cost benchmarking approaches.** Based on the new cost model, we need to find the best trade-off between computation and storage, which leads to the theoretical minimum application cost in the cloud. This minimum cost serves as important benchmark for evaluating the cost-effectiveness of storage strategies in the cloud. For different applications and users, cloud service providers should be able to provide benchmarking services according to their requirements. Hence benchmarking algorithms need to be investigated, so that we develop different benchmarking approaches to meet the requirements of different situations in the cloud.

3) **Cost-effective datasets storage strategies.** By investigating the trade-off between computation and storage, cost-effective storage strategies are needed for users to use in their applications at runtime in the cloud. Different from benchmarking, in practice, the minimum cost storage strategy may not be the best strategy for the applications in the cloud. First, storage strategies must be efficient enough to be facilitated at runtime in the cloud. Furthermore, users may have certain preferences on storing some particular datasets (e.g. tolerance of the accessing delay). Hence we need to design cost-effective storage strategies according to different requirements.
1.3 Overview of this Thesis

In particular, this thesis includes new concepts, solid theorems and complex algorithms, which form a suite of systematic and comprehensive solutions to deal with the issue of computation and storage trade-off in the cloud and bring cost-effectiveness to the applications for both users and cloud service providers. The thesis structure is depicted in Figure 1.1.

![Figure 1.1 Thesis structure](image-url)
In Chapter 2, we introduce the related work to this research. We start from introducing data management in some traditional scientific applications systems, especially in grid systems, and then we move to the cloud. By introducing some typical cloud systems for scientific application, we raise the issue of cost-effectiveness in the cloud. Next, we introduce some works that also touch the issue of computation and storage trade-off and analyse the differences to ours. At last, we introduce some works about data provenance which are the important foundation for our work.

In Chapter 3, we first introduce a motivating example which is a real world scientific application from astrophysics for searching pulsar in the universe. Based on this example we identify and analyse our research problems.

In Chapter 4, we first give a classification of the application data in the cloud and propose an important concept of Data Dependency Graph (DDG). DDG is built on data provenance which depicts the generation relationships of the datasets in the cloud. Based on DDG, we propose a new cost model for datasets storage in the cloud.

In Chapter 5, we develop novel minimum cost benchmarking approaches with algorithms for the best trade-off between computation and storage in the cloud. We propose two approaches, i.e. static on-demand benchmarking and dynamic on-the-fly benchmarking, to accommodate different application requirements in the cloud.

In Chapter 6, we develop innovative cost-effective storage strategies for user to facilitate at runtime in the cloud. According to different user requirements, we design different strategies accordingly, i.e. a highly efficient cost rate based strategy and a highly cost-effective local-optimisation based strategy.

In Chapter 7, we demonstrate experiment results to evaluate our work described in the entire thesis. First we introduce our cloud computing simulation environment, i.e. SwinCloud. Then we conduct general random simulations to evaluate the performance of our benchmarking approaches and storage strategies. At last, we demonstrate a case study of the pulsar searching application, in which all the research outcomes presented in this thesis are utilised.
Finally, in Chapter 8, we summarise the new ideas presented in this thesis, the major contributions of this research, and consequent further research works.

In order to improve the readability of this thesis, we put all proofs of theories, lemmas and corollaries in Appendix A and the notation index in Appendix B.
Chapter 2

Literature Review

This chapter reviews the existing literature related to this research. It is organised as follows. In Section 2.1, we summarise the data management work about scientific applications in the traditional distributed computing systems. In Section 2.2, we first review some existing work about deploying scientific applications in the cloud and raise the issue of cost-effectiveness, then we analyse some researches that have touched the issue of trade-off between computation and storage and point out the differences to our work. In Section 2.3, we introduce some work about data provenance which is the important foundation for our work.

2.1 Data Management of Scientific Applications in Traditional Distributed Systems

During the development of IT (Information Technology), e-science becomes more and more popular nowadays. Because scientific applications are often computation and data intensive, they are nowadays usually deployed in distributed systems to get the high performance computing resources and massive storage. Roughly speaking, one can make a distinction between two subgroups in the traditional distributed systems [11]: clusters (including the HPC system) and grids.

Early studies are in cluster computing systems [9]. Since cluster computing is a relative homogenous environment that has a tightly-coupled structure, data management in clusters is usually straightforward. The applications data are
commonly stored according to the system’s capacity and moved within the cluster via fast Ethernet connection while the applications execute.

Grid computing systems [40] are more heterogeneous than clusters. Given the similarity of grid and cloud [42], we mainly investigate the existing related work about grid computing system in this section. First we present some general data management technologies in grid. Then we investigate the data management in some grid workflow systems which are often utilised for running scientific applications. At last, we briefly introduce the data management technologies in some other distributed systems.

2.1.1 Data Management in Grid

Grid computing has many similarities with cloud computing [80, 83]. Both of them are heterogeneous computing environments for large scale applications. Data management technology in grid, Data Grid [28] in short, could be a valuable reference of cloud data management. Next, some important features of data grid are briefly summerised and some successful systems are also briefly introduced.

Data Grid [78] primarily deals with providing services and infrastructure for distributed data-intensive applications that need to access, transfer, and modify massive datasets stored in distributed storage resources. Generally speaking, it should have the following capabilities: (a) ability to search through numerous available datasets for the required dataset and to discover suitable data resources for accessing the data, (b) ability to transfer large-size datasets between resources as fast as possible, (c) ability for users to manage multiple copies of their data, (d) ability to select suitable computational resources and process data on them and (e) ability to manage access permissions for the data.

Grid technology was very popular in the late 1990s and early 2000s, because it is suitable for large scale computation and data intensive applications. Many data management systems were developed and gained great success. Some of the most successful ones are listed below and some of them have already been utilised in scientific applications.
**Grid DataFarm** [75] is a tightly-coupled architecture for storage in the grid environment. The architecture consists of nodes that have large disk space. Between the nodes there are interconnections via fast Ethernet. It also has a corresponding file system, process scheduler and parallel I/O APIs.

**GDMP** [72] mainly focuses on replication in the grid environment, which has been utilised in High Energy Physics. It uses the GridFTP technology to achieve high speed data transfer and provides point-to-point replication capability.

**GridDB** [58] builds an overlay based on relational database, and provides services for large scientific data analysis. It mainly focuses on the software architecture and query processing.

**SRB** [15] organises data into different virtual collections independent of their physical locations. It could provide a unified view of data files in the distributed environment. It is used in the Kepler workflow system.

**RLS (P-RLS)** [26, 23] maintains all the copies of data’s physical locations in the system, and provides data discovery services. Newly generated data could dynamically register in RLS, so that it could be discovered by the tasks. It has been used in Pegasus and Triana workflow systems.

**GSB** [79] is designed to mediate access to distributed resources. It could map tasks to resources and monitor task execution. GSB is the foundation of data management in the Gridbus workflow system.

**DaltOn** [51] is an infrastructure for Scientific Data Management. It supports the syntactic and semantic integration of data from multiple sources.

A comparison of these data management systems is listed in Table 2.1.

Although data grid has some similarities on data management of the cloud, they are essentially different. At the infrastructure level, grid systems are usually composed of several computing nodes built up with supercomputers, and the computing nodes are usually connected by fast Ethernet or dedicate networks, so that in data grid, efficient data management can be easily achieved with the high
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Table 2.1: A comparison of Data Grid
performance hardware. Cloud systems are based on the Internet and normally composed of data centres built up with commodity hardware, where data management is more challenging. More importantly, at the application level, most clouds are commercial systems while the grids are not. The wide utilisation of the pay-as-you-go model in the cloud makes the issue of cost-effectiveness more important than before.

2.1.2 Data Management in Grid Workflows

Scientific applications are typically very complex. They usually have a large number of tasks and need a long time for execution. Workflow technologies are important tools which can be facilitated to automate the executions of applications [34]. Many workflow management systems were developed in grid environments. Some of the most successful ones are listed below, as well as the features of their data management:

**Kepler** [61] is a scientific workflow management system in the grid environment. It points out that control-flow orientation and data-flow orientation are the difference between business and scientific workflows. Kepler has its own actor-oriented data modelling method that for large data in the grid environment. It has two Grid actors, called FileFetcher and FileStager, respectively. These actors make use of GridFTP [8] to retrieve files from, or move files to, remote locations on the Grid. In the runtime data management, Kepler adopts the SRB system [15].

**Pegasus** [33] is a workflow management system which mainly focuses on data-intensive scientific applications. It has developed some data management algorithms in the grid environment and uses the RLS [26] system as data management at runtime. In Pegasus, data are asynchronously moved to the tasks on demand to reduce the waiting time of the execution and dynamically delete the data that the task no longer needs to reduce the use of storage.

**Gridbus** [20] is grid toolkit. In this toolkit, the workflow system has several scheduling algorithms for the data-intensive applications in the grid environment based on a Grid Resource Broker [79]. The algorithms are designed based on different theories (GA, MDP, SCP, Heuristic), to adapt to different use cases.
**Taverna** [65] is a scientific workflow system for bioinformatics. It proposes a new process definition language, Sculf, which could model application data in a dataflow. It considers workflow as a graph of processors, each of which transfers a set of data inputs into a set of data outputs.

**MOTEUR** [44] workflow system advances Taverna’s data model. It proposes a data composition strategy by defining some specific operations.

**ASKALON** [84] is a workflow system designed for scheduling. It puts the computing overhead and data transfer overhead together to get a value “weight”. It does not discriminate the computing resource and data host. ASKALON also has its own process definition language called AGWL.

**Triana** [31] is a workflow system which is based on a problem-solving environment that enables the data-intensive scientific application to execute. For the grid, it has an independent abstraction middleware layer, called the Grid Application Prototype (GAP), enables users to advertise, discover and communicate with Web and peer-to-peer (P2P) services. Triana also uses the RLS to manage data at runtime.

**GridFlow** [54] is a workflow system which uses an agent-based system for grid resource management. It considers data transfer to computing resources and archive to storage resources as kinds of workflow tasks. But in this work, they do not discuss these data related workflow tasks.

In summary, for data management, all the workflow systems mentioned above have concerned the modelling of workflow data at build-time. Workflow data modelling is a long-term research topic in academia with matured theories, including Workflow Data Patterns [69], Dataflow Programming Language [53]. For data management at workflow run-time, most of these workflow systems simply adopt data management technology in data grid. They do not consider the dependencies among the application data. Only Pegasus proposes some strategies for workflow data placement based on dependency [27, 71], but they have not designed specific algorithms to achieve them. As all these workflow systems are in grid computing environment, they neither utilise the pay-as-you-as model nor investigate the issue of cost-effectiveness in deploying the applications.
2.1.3 Data Management in Other Distributed Systems

Many technologies are utilised for computation and data intensive scientific applications in distributed environments and have their own specialties. They could be importance references for our work. A brief overview is shown below [78]:

**Distributed Database** [68]. A distributed database (DDB) is a logically organised collection of data stored at different sites of a computer network. Each site has a degree of autonomy, which is capable of executing a local application, and also participates in the execution of a global application. A distributed database can be formed either by taking an existing single site database and splitting it over different sites (top-down approach) or by federating existing database management systems so that they can be accessed through a uniform interface (bottom-up approach). However, distributed databases are mainly designed for storing the structured data, which is not suitable for managing large generated datasets (e.g. raw data save in files) in scientific applications.

**Content Delivery Network** [38]. A Content Delivery Network (CDN) consists of a “collection of (non-origin) servers that attempt to offload work from origin servers by delivering content on their behalf”. That is, within a CDN, client requests are satisfied from other servers distributed around the Internet (also called edge servers) that cache the content originally stored at the source (origin) server. The primary aims of a CDN are, therefore, load balancing to reduce effects of sudden surges in requests, bandwidth conservation for objects such as media clips, and reducing the round-trip time to serve the content to the client. However, CDNs have not gained wide acceptance for data distribution because of the restricted model that they follow.

**Peer-to-Peer Network** [66]. The primary aims of a P2P network are to ensure scalability and reliability by removing the centralised authority, and also to ensure redundancy, to share resources, and to ensure anonymity. Such networks have mainly focused on creating efficient strategies to locate particular files within a group of peers, to provide reliable transfers of such files in the face of high volatility, and to manage high load caused by the demand for highly popular files. Currently,
major P2P content sharing networks do not provide an integrated computation and data distribution environment.

2.2 Cost-Effectiveness of Scientific Applications in the Cloud

Nowadays, scientific applications are often deployed in grid systems [61] because they have high performance and massive storage. However, building a grid system is extremely expensive and it is normally not open to other scientists around the world. When cloud computing was on horizon [37, 80, 83], it is deemed as the next generation of IT platforms that can deliver computing as a kind of utility [22]. Taking advantage of the new features, cloud computing technology has been utilised in many areas as soon as it is proposed, such as Data Mining [45], Database Application [17], Parallel Computing [46], Content Delivery [18] and so on.

2.2.1 Cost-Effectiveness of Deploying Scientific Applications in the Cloud

Scientific applications have already been introduced to the cloud and research on deploying applications in the cloud has become popular [29, 55, 57, 88, 81]. Cloud computing system for scientific applications, i.e. science cloud, has already commenced, where some successful and representative ones are as follows.

1. OpenNebula [5] project facilitates on-premise IaaS cloud computing, offering a complete and comprehensive solution for the management of virtualised data centres to enable private, public and hybrid clouds.

2. Nimbus Platform [4] is an integrated set of tools that deliver the power and versatility of infrastructure clouds to users. Nimbus Platform allows users to combine Nimbus, OpenStack, Amazon, and other clouds.

3. Eucalyptus [2] enables the creation of on-premise private clouds, with no requirements for retooling the organisation's existing IT infrastructure or need to introduce specialised hardware.
Foster et al. made a comprehensive comparison of grid computing and cloud computing [42], where two important differences related to this thesis are as follows:

1. Comparing to grid, cloud computing systems can provide the same high performance computing resources and massive storage required for scientific applications, but with a lower infrastructure construction cost among many other features. This is because cloud computing systems are composed of data centres which can be clusters of commodity hardware [83]. Hence, deploying scientific applications in the cloud could be more cost-effective than its grid counterpart.

2. By utilising the virtualisation technology, cloud computing systems are more scalable and elastic. Because new hardware can be easily added to the data centres, service providers can deliver cloud services based on the pay-as-you-go model and users can dynamically scale up or down the computation and storage resources they use.

Based on the new features of cloud, comparing to the traditional distributed computing systems like cluster and grid, a cloud computing system has a cost benefit from various aspects [12]. Assunção et al. [13] demonstrate that cloud computing can extend the capacity of clusters with a cost benefit. With Amazon clouds’ cost model and BOINC volunteer computing middleware, the work in [56] analyses the cost benefit of cloud computing versus grid computing. The work by Deelman et al. [36] also applies Amazon clouds’ cost model and demonstrates that cloud computing offers a cost-effective way to deploy scientific applications. In [49], Hoffa et al. conduct simulations of running an astronomy scientific workflow in cloud and clusters, which shows cloud scientific workflows are cost-effective. Meanwhile, Tsakalozos et al. [77] point out that by flexible utilisation of cloud resources, service provider’s profit can also be maximised. Especially, Cho et al. [30] further propose planning algorithms of how to transfer large bulks of scientific data to commercial clouds in order to run the applications.

The above works mainly focus on the comparison of cloud computing systems and the traditional distributed computing paradigms, which show that applications running in the cloud have cost benefits, but they do not touch the issue of computation and storage trade-off in the cloud.
2.2.2 Trade-Off between Computation and Storage in the Cloud

Based on the work introduced in Section 2.2.1, the research addressed in this thesis makes a significant step forward regarding the application cost in the cloud. We develop our approaches and strategies by investigating the issue of computation and storage trade-off in the cloud.

This research is mainly inspired by the work in two research areas: cache management and scheduling. With smart caching mechanism [39, 50, 52], system performance can be greatly improved. The similarity is that both pre-store some data for future use, while the difference is that caching is to reducing data accessing delay but our work is to reduce the application cost in the cloud. Works in scheduling focus on reducing various costs for either applications [82] or systems [86], but they investigate this issue from the perspective of resource provisioning and utilisation, not from the trade-off between computation and storage. In [43], Garg et al. investigate the trade-off between time and cost in the cloud, where users can reduce the computation time by using expensive CPU instances with higher performance. This trade-off is different to ours which aims to reduce the application cost in the cloud.

As the trade-off between computation and storage is an important issue, some researches have already embarked on this issue to a certain extent. Nectar system [48] is designed for automatic management of data and computation in data centres, where obsolete datasets are deleted and regenerated whenever reused in order to improve resource utilisation. In [36], Deelman et al. present that storing some popular intermediate data can save the cost in comparison to always regenerating them from the input data. In [7], Adams et al. propose a model to represent the trade-off of computation cost and storage cost, but have not given any strategy to find this trade-off.

In this thesis, for the first time, the issue of computation and storage trade-off for scientific datasets storage in the cloud is comprehensively and systematically investigated. We propose a new cost model to represent this trade-off, develop novel minimum cost benchmarking approaches to find the best trade-off [91, 89], and
design novel cost-effective datasets storage strategies based on this trade-off for users to store the application datasets [87, 92, 90].

2.3 Data Provenance in Scientific Applications

The research works on data provenance form an important foundation for our work. Data provenance is a kind of important metadata, in which the dependencies between application datasets are recorded [70]. The dependency depicts the generation relationship among the datasets. For scientific applications, data provenance is especially important because after the execution, some application datasets may be deleted, but sometimes the users have to regenerate them for either reuse or reanalysis [16]. Data provenance records the information of how the datasets were generated, which is very important for our research on the trade-off between computation and storage.

Due to the importance of data provenance in scientific applications, many works about recording data provenance of the system have been conducted [14, 47]. For example, some of them are for scientific workflow systems [14]. Some popular scientific workflow systems, such as Kepler [61], have their own system to record provenance during workflow execution [10]. Recently, research on data provenance in cloud computing systems has also appeared [63]. More specifically, Osterweil et al. [67] present how to generate a data derivation graph for the execution of a scientific workflow, where one graph records the data provenance of one execution, and Foster et al. [41] propose the concept of Virtual Data in the Chimera system, which enables automatic regeneration of datasets when needed.

2.4 Summary

In this chapter, the literatures of recent studies related to data management of scientific applications are reviewed. We start from the grid systems, and then move to the cloud. By investigating typical grid and cloud systems, we analyse the cost-effectiveness of deploying scientific applications in the cloud. Meanwhile, based on
In the literature review, we demonstrate that the core research issue of this thesis, i.e. computation and storage trade-off, is a significant yet barely touched issue in the cloud. At last, we introduce some works about data provenance which is an important foundation for our work.
Chapter 3
Motivating Example and Research Issues

The research in this thesis is motivated by a real world scientific application. In this chapter, Section 3.1 introduces a motivating example of pulsar searching application from Astrophysics; Section 3.2 analyses the problems and challenges of deploying scientific applications in the cloud; Section 3.3 describes the specific research issues of this thesis in detail.

3.1 Motivating Example

Swinburne Astrophysics group has been conducting pulsar searching surveys using the observation data from Parkes Radio Telescope, which is one of the most famous radio telescopes in the world\(^2\). Pulsar searching is a typical scientific application. It contains complex and time consuming tasks and needs to process terabytes of data. Figure 3.1 depicts a high level structure of the pulsar searching workflow, which is currently running on Swinburne high performance supercomputing facility\(^3\). There are three major steps in the pulsar searching process:

Figure 3.1 Pulsar searching workflow

1. Raw signal data recording. In Parkes Radio Telescope, there are 13 embedded beam receivers, by which signals from the universe are received. At the beginning, raw signal data are recorded at a rate of 1GB per second by the ATNF4 Parkes Swinburne Recorder5. Depending on different areas in the universe that the scientists want to conduct the pulsar searching survey, the observation time is normally from 4 minutes to one hour. The raw signal data are pre-processed by a local cluster at Parkes in real time and archived in tapes for permanent storage and future analysis.

2. Data preparation for pulsar seeking. The raw signal data recorded from the telescope have data from multiple beams interleaved, so at beginning of the workflow, different beam files are extracted from the raw data files and compressed. They are normally 1GB to 20GB each in size depending on the observation time. The scientists analyse the beam files to find the potentially contained pulsar signals. However, the signals are dispersed by the interstellar medium, where the scientists have to conduct a de-disperse step to counteract this effect. Since the potential dispersion source is unknown, a large number of de-dispersion files needs to be generated with different dispersion trials. For one dispersion trial of one beam file, the size of de-dispersion file is approximately 4.6MB to 80MB depending on the size of the input beam file (1GB to 20GB). In the current pulsar searching survey, 1200 is the minimum number of the dispersion trials, where this de-dispersion step takes 1 to 13 hours to finish and generate around 5GB to 90GB of de-dispersion files. Furthermore, for binary pulsar searching, every de-dispersion file needs a

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4 http://www.atnf.csiro.au/
5 http://astronomy.swin.edu.au/pulsar/?topic=apsr
separate *accelerate* step for processing. This step generates the accelerated de-dispersion files with the similar size in the *de-disperse* step.

3. Pulsar seeking. Based on the generated de-dispersion files, different seeking algorithms can be applied to search pulsar candidates, such as *FFT (Fast Fourier Transform) Seeking*, *FFA (Fast Fold Algorithm) Seeking*, and *Single Pulse Seeking*. For example, the *FFT Seeking* algorithm takes 7 to 80 minutes to seek the 1200 de-dispersion files with different size (5GB to 90GB). A candidate list of pulsars is generated after the seeking step which is saved in a text file, normally 1KB in size. Furthermore, by comparing the candidates generated from different beam files in a same time session, interference may be detected and some candidates may be *eliminated*. With the final pulsar candidates, we need to go back to the de-dispersion files to find their feature signals and *fold* them to XML files. Each candidate is saved in a separated XML file about 25KB in size. This step takes up to one hour depending on the number of candidates found in this searching process. At last, the XML files are visually displayed to scientists for *making decisions* on whether a pulsar has been found or not.

At present, all the generated datasets are deleted after having been used, and the scientists only store the raw beam data, which are extracted from the raw telescope data. Whenever there are needs of using the deleted datasets, the scientists will regenerate them based on the raw beam files. The generated datasets are not stored, mainly because the supercomputer is a shared facility that cannot offer sufficient storage capacity to hold the accumulated terabytes of data. However, some datasets are better to be stored. For example, the de-dispersion files can be more frequently used. Based on them, the scientists can apply different seeking algorithms to find potential pulsar candidates. For the large input beam files, the regeneration of the de-dispersion files will take more than 10 hours. It not only delays the scientists from conducting their experiments, but also requires a lot of computation resources. On the other hand, some datasets may need not to be stored. For example, the accelerated de-dispersion files, which are generated by the *accelerate* step, are not often used. The *accelerate* step is an optional step that is only for the binary pulsar searching. In light of this and given the large size of these datasets, they may be not
worth storing as it could be more cost effective to regenerate them from the de-
dispersion files whenever used.

3.2 Problem Analysis

Traditionally, scientific applications are normally deployed on the high performance computing facilities, such as clusters and grids. Scientific applications are often complex with huge datasets generated during their execution. How to store these datasets is often decided by the scientists who use the scientific applications. This is because the clusters and grids only serve for certain institutions. The scientists may store the datasets that are most valuable to them, based on the storage capacity of the system. However, for many scientific applications, the storage capacities are limited, such as the pulsar searching workflow introduced in Section 3.1. The scientists have to delete all the generated datasets because of the storage limitation. To store large scientific datasets, scientific communities have to set up data repositories [73] with large infrastructure investment. However, the storage bottleneck can be avoided in a cost-effective way if we deploy scientific applications in the cloud.

3.2.1 Requirements and Challenges of Deploying Scientific Applications in the Cloud

In a commercial cloud computing environment [1], theoretically, the system can offer unlimited storage resources. All the datasets generated by the scientific applications can be stored, if the users (e.g. scientists) are willing to pay for the required resources. However, new requirements and challenges also emerge for deploying scientific applications in the cloud, and whether to store the generated datasets or not is not an easy decision anymore.

1. All the resources in the cloud carry certain costs, so either storing or generating a dataset, we have to pay for the resources used. The application datasets vary in size, and have different generation costs and usage frequencies. Some of them may often be used whilst some others may be not. On one extreme, it is most likely not cost effective to store all the generated
datasets in the cloud. On the other extreme, if we delete them all, regeneration of frequently used datasets most likely imposes a high computation cost. We need a mechanism to balance the regeneration cost and the storage cost of the application data, in order to reduce the total application cost for dataset storage. This is also the core issue of this thesis, i.e. the trade-off between computation and storage.

2. The best trade-off between computation and storage cost may not be the best strategy for storing application data. When the deleted datasets are needed, the regeneration not only imposes computation cost, but also causes a time delay. Depending on the different time constraints of applications [25, 24], users’ tolerance of this computation may differ dramatically. Sometimes users may want the data to be available immediately, hence they would pay higher cost for storing some particular datasets; sometimes users do not care about waiting for it to become available, hence they may delete the seldom used dataset to reduce the overall application cost. Hence, we need to incorporate users’ preferences on data storage into this research.

3. The scientists cannot predict the usage frequencies of the application data anymore. For a single research group, if the data resources of the applications are only used by their own scientists, the scientists may estimate the usage frequencies of the datasets and decide whether to store or delete them. However, the cloud is normally not developed for a single scientist or institution, rather, for scientists from different institutions to collaborate and share data resources. Scientists from all over the world can easily visit the cloud via Internet to launch their applications, and all the application data are managed in the cloud. This requires data management to be automatic. Hence, we need to investigate the trade-off between computation and storage for all the users that can reduce the overall application cost. More specifically, the datasets usage frequencies should be discovered and obtained from the system logs, rather than manually set by the users. However, forecasting accurate datasets usage frequencies is out of this research’s scope and we list it as our future work in Section 8.3. In this thesis, we assume that the datasets usage frequencies be already obtained from the system logs.
3.2.2 Bandwidth Cost of Deploying Scientific Applications in the Cloud

Bandwidth is another common type of resource in the cloud. As cloud computing is such a fast growing market, more and more different cloud service providers will appear. In the future, we will be able to more flexibly select service providers to conduct our applications based on their pricing models. An intuitive idea is to incorporate different cloud service providers for applications, where we can store the data with one provider who has a lower price in storage resources, and choose another provider who has a lower price of computation resources to run the computation tasks. However, at present, normally it is not practical to run scientific applications across different cloud service providers because of the following reasons:

1. The data in scientific applications are often very large in size. They are too large to be transferred efficiently via the Internet. Due to bandwidth limitations of the Internet, in today’s scientific projects, delivery of hard disks is a common practice to transfer application data, and it is also considered to be the most efficient way to transfer, say, terabytes of data [12]. Nowadays, express delivery companies can deliver the hard disks nation-wide by the end of the next day and world-wide in 2 or 3 days. In contrast, transferring one terabyte data via the Internet would take more than 10 days at a speed of 1MB/s. To break the bandwidth limitation, some institutions set up dedicated optic fibres to transfer data. For example, Swinburne University of Technology has built a dedicated fibre to Parkes telescope station with gigabit bandwidth. However, it is mainly used for transferring gigabytes of data. To transfer terabytes, or petabytes, of data, scientists would still prefer to ship hard disks. Furthermore, building (dedicated) fibre connections is very expensive, and they are not yet widely used in the Internet. Hence, transferring scientific application data between different cloud service providers via the Internet is not efficient.

2. Cloud service providers place high price on data transfer in and out their data centres. In contrast, data transfers within one cloud service provider’s data centres are usually free. For example, the data transfer price of Amazon cloud
service is: $0.12 per GB\textsuperscript{6} of data transferred out. Comparing to the storage price of $0.15 per GB per month\textsuperscript{7}, the data transfer price is relatively high, so that finding a cheaper storage cloud service provider and transferring data may not be cost effective. In cloud service providers’ position, they charge high price on data transfer not only because of the bandwidth limitation, but also as a business strategy. As data are deemed as an important resource today, cloud service providers want users to keep all the application data in their storage cloud. For example, Amazon places a zero price on data transferred into its data centres, which means users could upload their data to Amazon’s cloud storage for free. However, the price of data transferred out of Amazon is not only not free, but also rather expensive.

Due to the reasons above, we assume that the scientists only utilise cloud services from one service provider to deploy their applications. Furthermore, according to some researches [36, 49], the cost-effective way of doing science in the cloud is to upload all the application data to the cloud storage and run all the applications with the cloud services. So we assume that the scientists upload all the original data to the cloud to conduct their processing. Hence the cost of transferring data in and out of the cloud only depends on the applications themselves (i.e. how much original and result data the applications have), and has no impact on the usage of computation and storage resources for running the applications in the cloud. Hence, we do not incorporate data transfer cost in the trade-off between computation and storage at this stage.

\section*{3.3 Research Issues}

In this section, we discuss the research issues tackled in this thesis based on the problems analysed in Section 3.2.

\textsuperscript{6} \url{http://aws.amazon.com/ec2/pricing/} - The prices may fluctuate from time to time according to market factors.

\textsuperscript{7} \url{http://aws.amazon.com/s3/pricing/} - The prices may fluctuate from time to time according to market factors.
3.3.1 Cost Model for Datasets Storage in the Cloud

In a commercial cloud, in theory, users can get unlimited resources for both computation and storage. However, they are responsible for the cost of the resources used due to the pay-as-you-go model. Hence, users need a new and appropriate cost model that can represent the cost that they actually incur on their applications in the cloud.

For the large generated application datasets in the cloud, users can be given the choice to store them for future use or delete them for saving the storage cost. Different storage strategies lead to different consumptions of storage and computation resources and finally lead to different total application costs. Furthermore, because there are dependencies among the application datasets, i.e. computation task can operate on one or more datasets and generate new one(s), the storage status of a dataset is not only dependent on the generation cost and storage cost of itself, but also dependent on the storage status of its predecessors and successors. The new cost model should be able to represent the total cost of the applications based on the trade-off between computation and storage in the cloud, where data dependencies are taken into account.

3.3.2 Minimum Cost Benchmarking Approaches

Minimum cost benchmarking is to find the theoretical minimum application cost based on the cost model, which is also the best trade-off between computation and storage in the cloud. Due to the pay-as-you-go model in the cloud, cost is one of the most important factors that users care about. As a rapidly increasing number of datasets is generated and stored in the cloud, users need to evaluate the cost effectiveness of their storage strategies. Hence the service providers should be able and need to provide benchmarking services that can inform the minimum cost of storing the application datasets in the cloud.

Calculating the minimum cost benchmark is a seemingly NP-hard problem, because there are complex dependencies among the datasets in the cloud. Furthermore, this application cost in the cloud is of a dynamic value. This is because
of the dynamic nature of the cloud computing system, i.e. 1) new datasets may be generated in the cloud at any time; and 2) the usage frequencies of the datasets may also change as time goes on. Hence, the minimum cost benchmark may change from time to time. In order to guarantee the quality of service (QoS) in the cloud, there should be different benchmarking approaches accommodating different situations. For example, in some applications, users may only need to know the benchmark before or occasionally during application execution. In this situation, benchmarking should be provided as a static service which can respond to users’ requests on-demand. However, in some applications, users may have more frequent benchmarking requests at runtime. In this situation, benchmarking should be provided as a dynamic service which can respond to users’ requests on the fly.

3.3.3 Cost-Effective Storage Strategies

Based on the trade-off between computation and storage, cost-effective storage strategies need to be designed in this thesis. Different from benchmarking, in practice, the minimum cost storage strategy may not be the best strategy for the applications, because storage strategies are for users to use at runtime in the cloud and should take users’ preferences into consideration.

Beside cost-effectiveness, storage strategies must be efficient enough to be facilitated at runtime in the cloud. For different applications, the requirements of efficiency may be different. On one hand, some applications may need highly efficient storage strategies with acceptable though not optimal cost-effectiveness. On the other hand, some applications may need highly cost-effective storage strategies with acceptable efficiency. According to different requirements, we need to design corresponding storage strategies.

Furthermore, to reflect users’ preferences on the datasets storage, we need to incorporate related parameters into the strategies which 1) guarantee all the application datasets’ regenerations can fulfill users’ tolerance of data accessing delay, and 2) allow users to store some datasets according to their preferences.
3.4 Summary

In this chapter, based on a real world pulsar searching scientific application from astrophysics, we analyse the requirements of data storage in scientific applications and how cloud computing systems can fulfill these requirements. Then we analyse the problems of deploying scientific applications in the cloud and define the scope of this research. Based on the analysis, we present the detailed research issues of this thesis: 1) cost model for datasets storage in the cloud; 2) minimum cost benchmarking approaches; and 3) practical datasets storage strategies.
Chapter 4
Cost Model of Datasets Storage in the Cloud

In this section, we present our new cost model of datasets storage in the cloud. Specifically, Section 4.1 introduces a classification of application data in the cloud and further expresses the scope of this research. Section 4.2 introduces data provenance and describes the concept of DDG (Data Dependency Graph) which is used to depict the data dependencies in the cloud. Based on Sections 4.1 and 4.2, in Section 4.3 we describe the new cost model and its important attributes in detail.

This cost model has been utilised in our work presented in [87, 92, 90, 91, 89].

4.1 Classification of Application Data in the Cloud

In general, there are two types of data stored in the cloud storage, original data and generated data:

1. *Original data* are the data uploaded by users, and in scientific applications they are usually the raw data collected from the devices in the experiments. In the cloud, they are the initial input of the applications for processing and analysis. The most important feature of these data is that if they are deleted, they cannot be regenerated by the system.
2. *Generated data* are the data produced in the cloud computing system while the applications run. They are the intermediate or final computation results of the application which can be used in the future. The most important feature of these data is that they can be regenerated by the system and more efficiently if we know their provenance.

For *original data*, only the users can decide whether they should be stored or deleted, since they cannot be regenerated once deleted. Hence, our research only focuses on generated data in the cloud that the system can automatically decide their storage status for achieving the best trade-off between computation and storage. In this thesis, we refer generated data as dataset(s).

### 4.2 Data Provenance and Data Dependency Graph (DDG)

Scientific applications have many computation and data intensive tasks that generate many datasets of considerable size. There exist dependencies among these datasets, which depict the generation (in another word, derivation) relationships. For scientific applications, after the execution, some datasets may be deleted, but if so, sometimes they need to be regenerated for either reuse or reanalysis [16]. To regenerate a dataset in the cloud, we need to find its stored predecessors and start the computation from them. Hence the regeneration of a dataset includes not only the computation of the dataset itself, but also the regeneration of its deleted predecessors, if any. This makes minimising the total application cost a very complex problem.

Data provenance is a kind of important metadata which records the dependencies among datasets [70], i.e. the information of how the datasets were generated. Data provenance is especially important for scientific applications in the cloud, because the regeneration of datasets from the original data may be very time consuming, and therefore carry a high cost. With data provenance information, the regeneration of the requested dataset could start from some stored (predecessor) datasets, hence more efficient and cost effective.

Taking the advantage of data provenance, we can build a DDG. All the datasets once generated (or modified) in the cloud, whether stored or deleted, their
references are recorded in the DDG as different nodes. In DDG, every node denotes a dataset. Figure 4.1 shows a simple DDG, where every node in the graph denotes a dataset. Dataset \( d_1 \) pointing to dataset \( d_2 \) means that \( d_1 \) is used to generate \( d_2 \); and \( d_2 \) pointing to \( d_3 \) and \( d_5 \) means that \( d_2 \) is used to generate \( d_3 \) and \( d_5 \) based on different operations; datasets \( d_4 \) and \( d_6 \) pointing to dataset \( d_7 \) means that \( d_4 \) and \( d_6 \) are used together to generate \( d_7 \).

![Figure 4.1 A simple Data Dependency Graph (DDG)](image)

DDG is a directed acyclic graph (DAG). This is because DDG records the provenances of how datasets are derived in the system as time goes on. In other words, it depicts the generation relationships of datasets. When some of the deleted datasets need to be reused, in general, we need not regenerate them from the original data. With DDG, the system can find the predecessors of the requested dataset, so that they can be regenerated from their nearest stored predecessors.

We denote a dataset \( d_i \) in DDG as \( d_i \in DDG \), and to better describe the relationships of datasets in DDG, we define two symbols \( \rightarrow \) and \( \leftrightarrow \):

- \( \rightarrow \) denotes that two datasets have a generation relationship, where \( d_i \rightarrow d_j \) means that \( d_i \) is a predecessor dataset of \( d_j \) in the DDG. For example, in the DDG depicted in Figure 4.1, we have \( d_1 \rightarrow d_2, d_1 \rightarrow d_4, d_5 \rightarrow d_7, d_1 \rightarrow d_7 \), etc. Furthermore, \( \rightarrow \) is transitive, i.e.

  \[
  d_i \rightarrow d_j \rightarrow d_k \iff d_i \rightarrow d_j \land d_j \rightarrow d_k \Rightarrow d_i \rightarrow d_k.
  \]

- \( \leftrightarrow \) denotes that two datasets do not have a generation relationship, where \( d_i \leftrightarrow d_j \) means that \( d_i \) and \( d_j \) are in different branches in DDG. For example, in the DDG depicted in Figure 4.1, we have \( d_1 \leftrightarrow d_5, d_1 \leftrightarrow d_6 \), etc. Furthermore, \( \leftrightarrow \) is commutative, i.e. \( d_i \leftrightarrow d_j \iff d_j \leftrightarrow d_i \).
4.3 Datasets Storage Cost Model in the Cloud

In a commercial cloud computing environment, if the users want to deploy and run applications, they need to pay for the resources used. The resources are offered by cloud service providers, who have their cost models to charge the users on storage and computation. For example, one set of Amazon cloud services’ prices is as follows:

- $0.15 per Gigabyte per month for the storage resources;
- $0.1 per CPU instance hour for the computation resources;

In this thesis, in order to represent the trade-off between computation and storage, we define the total cost for running a scientific application in the cloud as follows:

\[ \text{Cost} = \text{Computation} + \text{Storage}, \]

where the total cost of the application, Cost, is the sum of Computation, which is the total cost of computation resources used to regenerate datasets, and Storage, which is the total cost of storage resources used to store the datasets. As indicated in Section 4.1, our research only focuses on the generated data. The total application cost in this thesis does not include computation cost of the application itself and the storage cost of the original data.

To calculate the total application cost in the cloud, we define some important attributes for the datasets in DDG. For dataset \( d_i \), its attributes are denoted as: \( <x_i, y_i, f_i, v_i, provSet_i, CostR_i> \), where

- \( x_i \) denotes the generation cost of dataset \( d_i \) from its direct predecessors. To calculate this generation cost, we have to multiply the time of generating dataset \( d_i \) by the price of computation resources. Normally the generation time can be obtained from the system logs.

---

8 The prices may fluctuate from time to time according to market factors.
• $y_i$ denotes the cost of storing dataset $d_i$ in the system per time unit (i.e. storage cost rate). This storage cost rate can be calculated by multiplying the size of dataset $d_i$ and the price of storage resources per time unit.

• $f_i$ is a flag, which denotes the status whether this dataset is stored or deleted in the system.

• $v_i$ denotes the usage frequency, which indicates how often $d_i$ is used. In cloud computing systems, datasets may be shared by many users from the Internet. Hence $v_i$ cannot be defined by a single user and should be an estimated value from $d_i$’s usage history recorded in the system logs.

• $provSet_i$ denotes the set of stored provenances that are needed when regenerating dataset $d_i$. In other words, it is the set of references of stored predecessor datasets that are adjacent to $d_i$ in the DDG. If we want to regenerate $d_i$, we have to find its direct predecessors, which may also be deleted, so we have to further find the stored predecessors of $d_i$. $provSet_i$ is the set of the nearest stored predecessors of $d_i$ in the DDG. Figure 4.2 shows the $provSets$ of a dataset in different situations. Formally, we can describe dataset $d_i$’s $ProvSet_i$ as follows:

$$provSet_i = \{d_j | \forall d_j \in DDG \land f_j = "stored" \land d_j \rightarrow d_i \land \left( \exists d_k \in DDG \land d_j \rightarrow d_k \rightarrow d_i \land f_k = "deleted" \right) \}$$

$provSet_i$ is a very important attribute of a dataset in calculating its generation cost. When we want to regenerate a dataset in DDG, we have to start the computation from the dataset in its $provSet$. Hence, for dataset $d_i$, its generation cost is:

$$genCost(d_i) = x_i + \sum_{d_j | d_j \in provSet_i \land d_j \rightarrow d_k \rightarrow d_i} x_k$$  \hspace{1cm} (4.1)$$

This cost is a total cost of 1) the generation cost of dataset $d_i$ from its direct predecessor datasets and 2) the generation costs of $d_i$’s deleted predecessors that need to be regenerated as well.
Figure 4.2 A dataset’s provSets in a DDG in different situations

- **CostR**: is \(d_i\)'s cost rate, which means the average cost per time unit of the dataset \(d_i\) in the system. If \(d_i\) is a stored dataset, then \(CostR_i = y_i\). If \(d_i\) is a deleted dataset in the system, when we need to use \(d_i\), we have to regenerate it. So we multiply the generation cost of \(d_i\) by its usage frequency and use this value as the cost rate of \(d_i\) in the system, i.e. \(CostR_i = genCost(d_i) \times v_i\). The storage statuses of the datasets have significant impacts on their cost rates. Formally, dataset \(d_i\)'s cost rate is:

\[
CostR_i = \begin{cases} 
y_i, & f_i = \text{"stored"} 
genCost(d_i) \
y_i, & f_i = \text{"deleted"} \end{cases}
\]  
\tag{4.2}

Based on the definition of the attributes above, we can calculate the total cost rate of storing the datasets recorded in a DDG, which is \(\sum_{d_i \in DDG} CostR_i\). This cost rate is the cost of computation and storage resources consumption in the cloud per time unit, which is also the cost of running the application in the cloud per time unit. Given a time duration \(t\), the total application cost of storing the datasets recorded in a DDG is the integral of the cost rate in this duration as a function of time \(t\), which is

\[
Total\ Cost = \int_t \left( \sum_{d_i \in DDG} CostR_i \right) \cdot dt
\]  
\tag{4.3}

We further define the storage strategy of a DDG as \(S\), where \(S\) is a set of datasets in the DDG denoted as \(S \subseteq DDG\), which means storing the datasets in \(S\) in the cloud and deleting the rest. We denote the sum of cost rates of storing the datasets recorded in a DDG with the storage strategy \(S\) as \(SCR\), formally:

\[
SCR = \left\{ \sum_{d_i \in DDG} CostR_i \right\}_S
\]  
\tag{4.4}
Based on the definition above, different storage strategies lead to different cost rates (i.e. cost per time unit) for the application in the cloud. This cost rate is the total consumption of computation and storage resources in the cloud per time unit, hence represents the trade-off between computation and storage. Our work aims at minimising this cost rate so that we can help both service providers and users to reduce the application cost in the cloud.

### 4.4 Summary

In this chapter, we first introduce a classification of the application data in the cloud, i.e. original data and generated data, and further point out that our research only focuses on the generated data. Then we describe the concept of Data Dependency Graph (DDG), which is a very important for datasets storage in the cloud. At last, we present the cost model of datasets storage based on DDG, where the total application cost is the sum of the computation cost for regenerating datasets and the storage cost for storing datasets. Furthermore, we use a cost rate (i.e. total consumption of computation and storage resources in the cloud per time unit) to represent the trade-off between computation and storage. By minimising this cost rate, our work presented later aims at cutting the application cost in the cloud.
Chapter 5
Minimum Cost Benchmarking Approaches

In this chapter, we present our minimum cost benchmarking approaches for the applications in the cloud. The benchmark is the theoretical minimum application cost in the cloud, which is also the best trade-off between computation and storage. As introduced in Section 4.3, we use a cost rate to represent this trade-off (i.e. SCR). Benchmarking is to find the minimum value of this cost rate (i.e. the SCR with the minimum cost storage strategy of the DDG). Due to the complex dependencies among the datasets in the cloud, the DDG is a Directed Acyclic Graph (DAG). Hence, calculating the minimum cost benchmark is a seemingly NP-hard problem based on the cost model introduced in Section 4.3. Furthermore, the application cost in the cloud is of a dynamic value. This is because of the dynamic nature of the cloud computing system, i.e. 1) new datasets may be generated in the cloud at any time; and 2) the usage frequencies of the datasets may also change as time goes on. Hence, the minimum cost benchmark may change from time to time. In this chapter, we present two benchmarking approaches: one static and one dynamic.

Section 5.1 presents a novel static on-demand minimum cost benchmarking approach. This approach is suitable for the situation that no frequent benchmarking is requested. In this situation, the benchmarking should be provided as an on-demand service. Whenever a benchmarking request comes, the corresponding algorithms will be triggered to calculate the minimum cost benchmark, which is one time only.
computation based on the current status of the application. This section is mainly based on our work presented in [91].

Section 5.2 presents a novel dynamic on-the-fly minimum cost benchmarking approach. This approach is suitable for the situation that more frequent benchmarking is requested at runtime. In this approach, by saving and utilising the pre-calculated results, whenever the application cost changes in the cloud, we can quickly calculate the new minimum cost benchmark. By keeping the benchmark dynamically updated, benchmarking requests can be instantly responded on the fly. This section is mainly based on our work presented in [89].

5.1 Static On-Demand Minimum Cost Benchmarking Approach

In this section, we present our on-demand minimum cost benchmarking approach. Specifically, we describe the novel design of a Cost Transitive Tournament Shortest Path (CTT-SP) based algorithm that can find the Minimum Cost Storage Strategy (MCSS) for a given DDG. The basic idea of the CTT-SP algorithm is to construct a Cost Transitive Tournament (CTT) based on the DDG. In a CTT, we guarantee that the paths from the start dataset to the end dataset have a one-to-one mapping to the storage strategies of the DDG, and the length of every path equals to the total cost rate of the corresponding storage strategy. Then we can use the well-known Dijkstra shortest path algorithm (or Dijkstra algorithm for short) to find the Shortest Path (SP) in the CTT, which in fact represents the MCSS, and the cost rate of the MCSS (i.e. SCR) is the minimum cost benchmark.

To describe the approach in detail, in Section 5.1.1 we start with the CTT-SP algorithm for the linear DDG, and then in Section 5.1.2 we expand it to the DDG with one block, next in Section 5.1.3 we present the general CTT-SP algorithm for on-demand benchmarking. The experiment results are presented in Chapter 7, jointly along with others.
5.1.1 CTT-SP Algorithm for Linear DDG

Linear DDG means a DDG with no branches, where each dataset in the DDG only has one direct predecessor and successor except the first and last datasets.

Given a linear DDG, which has datasets \(\{d_1, d_2 \ldots d_n\}\). The CTT-SP algorithm has the following four steps:

**Step 1:** We add two virtual datasets in the DDG, \(d_s\) before \(d_1\) and \(d_e\) after \(d_n\), as the start and end datasets, and set \(x_s = y_s = 0\) and \(x_e = y_e = 0\).

**Step 2:** We add new directed edges in the DDG to construct the transitive tournament. For every dataset in the DDG, we add edges that start from it and point to all its successors. Formally, for dataset \(d_i\), it has out-edges to all the datasets in the set of \(\{d_j | d_j \in DDG \land d_i \rightarrow d_j\}\), and in-edges from all the datasets in the set of \(\{d_k | d_k \in DDG \land d_k \rightarrow d_i\}\). Hence, for any two datasets \(d_i\) and \(d_j\) in the DDG, we have an edge between them, denoted as \(e < d_i, d_j >\). Formally,

\[
d_i, d_j \in DDG \land d_i \rightarrow d_j \Rightarrow \exists e < d_i, d_j >
\]

**Step 3:** We set weights to the edges. The reason we call the graph Cost Transitive Tournament is because the weights of its edges are composed of the cost rates of datasets. For an edge \(e < d_i, d_j >\), we denote its weight as \(\omega < d_i, d_j >\), which is defined as the sum of cost rates of \(d_j\) and the datasets between \(d_i\) and \(d_j\), supposing that only \(d_i\) and \(d_j\) are stored and the rest of datasets between \(d_i\) and \(d_j\) are all deleted. Formally:

\[
\omega < d_i, d_j > = \text{CostR}_j + \sum_{\{d_k | d_k \in DDG \land d_i \rightarrow d_k \rightarrow d_j\}} \text{CostR}_k
\]

\[
= y_j + \sum_{\{d_k | d_k \in DDG \land d_i \rightarrow d_k \rightarrow d_j\}} \left( \genCost(d_k) * v_k \right)
\]

(5.1)

Since we are discussing the linear DDG, for the datasets between \(d_i\) and \(d_j\), \(d_i\) is the only dataset in their provSets. Hence we can further derive:

\[
\omega < d_i, d_j > = y_j + \sum_{\{d_k | d_k \in DDG \land d_i \rightarrow d_k \rightarrow d_j\}} \left( x_k + \sum_{\{d_j | d_j \in DDG \land d_i \rightarrow d_j \rightarrow d_k\}} x_k \right) * v_k
\]
In Figure 5.1, we demonstrate a simple example of constructing CTT for a DDG that has three datasets \((d_1, d_2, d_3)\), where \(d_s\) is the start dataset that only has out-edges and \(d_e\) is the end dataset that only has in-edges.

**Figure 5.1 An example of constructing CTT**

**Step 4:** We find the shortest path of CTT. From the construction steps, we can clearly see that the CTT is an acyclic complete oriented graph. Hence we can use the Dijkstra algorithm to find the shortest path from \(d_s\) to \(d_e\). The Dijkstra algorithm is a classic greedy algorithm to find the shortest path in graph theory. We denote the shortest path from \(d_s\) to \(d_e\) as \(P_{\text{min}}<d_s, d_e>\).

**Theorem 5.1:** Given a linear DDG with datasets \(\{d_1, d_2 \ldots d_n\}\), the length of \(P_{\text{min}}<d_s, d_e>\) of its CTT is the minimum cost rate for storing the datasets in the DDG, and the corresponding storage strategy is to store the datasets that \(P_{\text{min}}<d_s, d_e>\) traverses.

Theorem 5.1 demonstrates that the linear CTT-SP algorithm finds the MCSS of linear DDGs, hence can be used for minimum cost benchmarking. Figure 5.2

---

9 As indicated at the end of Chapter 1, proofs of all the theorems, lemmas and corollaries are in Appendix A of this thesis.
shows the pseudo-code of the linear CTT-SP algorithm. To construct the CTT, we first create the cost edges (lines 1-3), and then calculate their weights (lines 4-11). Next, we use the Dijkstra algorithm to find the shortest path (line 12), and return the MCSS and the minimum cost benchmark (lines 13-15).

Algorithm: Linear CTT-SP
Input: Start dataset \(d_s\); End dataset \(d_e\);
A linear DDG; //include \(d_s\) and \(d_e\)
Output: \(S\) //MCSS of the DDG
\(SCR\) //Minimum cost benchmark

\[
\begin{align*}
01. & \text{for ( every dataset } d_i \text{ in DDG )} \quad \text{//Create CTT} \\
02. & \quad \text{for ( every dataset } d_j, \text{ where } d_i \rightarrow d_j) \quad \text{//Create an edge} \\
03. & \quad \quad \text{weight} = 0; \quad \text{//Create an edge} \\
04. & \quad \text{for ( every dataset } d_k, \text{ where } d_i \rightarrow d_k) \quad \text{//Calculate the weight of an edge} \\
05. & \quad \quad \text{genCost} = 0; \quad \text{//Calculate the weight of an edge} \\
06. & \quad \quad \text{for ( every dataset } d_h, \text{ where } d_k \rightarrow d_h) \quad \text{//Accumulate generation cost rate} \\
07. & \quad \quad \quad \text{weight} = \text{weight} + (x_h + \text{genCost}) \cdot y_j; \quad \text{//Accumulate generation cost rate} \\
10. & \quad \text{weight} = \text{weight} + y_j; \quad \text{//Set weight to an edge} \\
11. & \quad \text{Set } \omega < d_i, d_j >= \text{ weight}; \quad \text{//Set weight to an edge} \\
12. & \quad \text{ } P_{\text{max}} = \text{Dijkstra } (d_s, d_e, \text{ CTT}); \quad \text{//Find the shortest path} \\
13. & \quad \text{ } S = \text{ set of datasets that } P_{\text{max}} \text{ traversed; } \text{//Except } d_s \text{ and } d_e \\
14. & \quad \text{ } \text{SCR} = \left(\sum_{d \in \text{DDG}} \text{CostR} \right) / S; \quad \text{//Minimum cost benchmark} \\
15. & \text{Return } S, \text{SCR};
\end{align*}
\]

Figure 5.2 Pseudo-code of linear CTT-SP algorithm for benchmarking

From the pseudo-code in Figure 5.2, we can clearly see that for a linear DDG with \(n\) datasets, we have to add a magnitude of \(n^2\) edges to construct the CTT (line 3 with two nested loops in lines 1-2), and for the longest edge, the time complexity of calculating its weight is also \(O(n^2)\) (lines 5-11 with two nested loops), so a total of \(O(n^4)\). Next, the Dijkstra algorithm (line 12) has the known time complexity of \(O(n^2)\). Hence the linear CTT-SP algorithm has a worst case time complexity of \(O(n^4)\).

5.1.2 Minimum Cost Benchmarking Algorithm for DDG with One Block

Linear DDG is a special case of general DDGs. In the real world, application datasets generated in the cloud may have complex relationships, such that different datasets may be generated from a single dataset by different operations, and different datasets may be used together to generate one dataset. In other words, DDG may have
branches, where the linear CTT-SP algorithm introduced in Section 5.1.1 cannot be directly applied. This is because current CTT can only be constructed on linear DDG, which means that the datasets in a DDG must be totally ordered. In this sub-section, we discuss how to find the MCSS of the DDG that has a sub-branch within one block for benchmarking.

5.1.2.1 Constructing CTT for DDG with one block

First we introduce the concept of “block” in DDG. Block is a set of sub-branches in the DDG that splits from a common dataset and merges into another common dataset. We denote the block as $B$. Figure 5.3 shows a DDG with a simple block $B=\{d_3, d_4, d_5, d_6\}$, we will use it as the example to illustrate the construction of CTT in our new algorithm.

![Figure 5.3 An example of constructing CTT for DDG with one block](image)

To construct the CTT, we need the datasets in DDG to be totally ordered. Hence, for the DDG with one block, we only choose one branch to construct the CTT, as shown in Figure 5.3. We call the linear datasets which are chosen to construct the
CTT “main branch”, denoted as $MB$, and call the rest of datasets “sub-branch(es)”, denoted as $SB$. For example, in Figure 5.3’s DDG, $MB=\{d_1, d_2, d_5, d_6, d_7, d_8\}$ and $SB=\{d_3, d_4\}$. Due to the existence of the block, the edges can be classified into four categories. The definition of this classification is as follows:

- **in-block edge**: $e<d_1, d_j>$ is an in-block edge meaning that the edge starts from $d_i$, which is a dataset outside of the block, and points to $d_j$, which is a dataset in the block, such as $e<d_2, d_5>$, $e<d_1, d_6>$ in Figure 5.3. Formally, we define $e<d_1, d_j>$ as an in-block edge, where

$$\exists d_k \in DDG \land d_i \rightarrow d_k \land d_j \leftrightarrow d_k .$$

- **out-block edge**: $e<d_1, d_j>$ is an out-block edge meaning that the edge starts from $d_i$, which is a dataset in the block, and points to $d_j$, which is a dataset outside of the block, such as $e<d_6, d_7>$, $e<d_5, d_8>$ in Figure 5.3. Formally, we define $e<d_1, d_j>$ as an out-block edge, where

$$\exists d_k \in DDG \land d_i \leftrightarrow d_k \land d_k \rightarrow d_j .$$

- **over-block edge**: $e<d_1, d_j>$ is an over-block edge meaning that the edge crosses over the block, where $d_i$ is a dataset preceding the block, $d_j$ is a dataset succeeding the block, such as $e<d_2, d_7>$, $e<d_1, d_8>$ in Figure 5.3. Formally, we define $e<d_1, d_j>$ as an over-block edge, where

$$\exists d_k, d_h \in DDG \land d_i \leftrightarrow d_k \land d_i \rightarrow d_h \rightarrow d_j \land d_i \rightarrow d_k \rightarrow d_j .$$

- **ordinary edge**: $e<d_1, d_j>$ is an ordinary edge meaning that datasets between $d_i$ and $d_j$ are totally ordered, such as $e<d_3, d_2>$, $e<d_5, d_6>$, $e<d_7, d_8>$ in Figure 5.3. Formally, we define $e<d_1, d_j>$ as an ordinary edge, where

$$\neg \exists d_k \in DDG \land \left(\left( d_i \rightarrow d_k \land d_k \leftrightarrow d_j \right) \lor \left( d_i \leftrightarrow d_k \land d_k \rightarrow d_j \right) \lor \left( d_h \in DDG \land d_h \leftrightarrow d_k \land d_h \rightarrow d_j \rightarrow d_i \rightarrow d_k \rightarrow d_j \right) \lor \left( d_h \in DDG \land d_h \leftrightarrow d_k \land d_h \rightarrow d_j \rightarrow d_i \rightarrow d_k \rightarrow d_j \right) \right) .$$
5.1.2.2 Setting weights to different types of edges

The essence of the CTT-SP algorithm is the rules for setting weights to the cost edges. In order to set weights to different types of edges in the DDG with one block, we need to introduce an important corollary of Theorem 5.1.

**Corollary 5.1:** During the process of finding the shortest path, for every dataset $d_f$ that is discovered by the Dijkstra algorithm, we have a path $P_{min}<d_s, d_f>$ from $d_s$ to $d_f$ and a set of datasets $S_f$ that $P_{min}<d_s, d_f>$ traverses. $S_f$ is the MCSS of the sub DDG segment $\{d_i|d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f\}$.

In the CTT-SP algorithm, the rules for setting weights to the edges guarantee that the paths from the start dataset $d_s$ to every dataset $d_i$ in the CTT represent the storage strategies of the datasets $\{d_k|d_k \in DDG \land d_s \rightarrow d_k \rightarrow d_i\}$, and Corollary 5.1 further indicates that the shortest path represent the MCSS. As defined in Section 5.1.1, the weight of the edge $e<d_i, d_f>$ is the sum of cost rates of $d_j$ and the datasets between $d_i$ and $d_j$, supposing that only $d_i$ and $d_j$ are stored and the rest of datasets between $d_i$ and $d_j$ are all deleted. In the DDG with one block, this rule is still applicable to the ordinary edges and in-block edges.

However, if $e<d_i, d_f>$ is an out-block edge or over-block edge, formula (5.1) in Section 5.1.1 is not applicable for calculating its weight anymore, because of the following reasons.

1) Due to the existence of the block, the datasets succeeding the block may have more than one datasets in their provSets. The generation of these datasets needs not only $d_i$, but also the stored provenance datasets from the other sub-branches of the block. For example, according to formula (5.1) in Section 5.1.1, the weight of the out-block edge $e<d_5, d_8>$ in Figure 5.3 is

$$\omega <d_5,d_8> = y_8 + genCost(d_8)*v_6 + genCost(d_7)*v_7,$$

where if we want to calculate $genCost(d_7)$, we also have to know the storage statuses of $d_3$ and $d_4$. The same problem also exists when calculating the weights of the over-
block edges. Hence, to calculate the weights of out-block and over-block edges, we have to know the storage strategies of all the sub-branches in the block.

2) The path from $d_s$ to $d_j$ cannot represent the storage strategy of all the datasets $\{d_k | d_k \in DDG \land d_s \rightarrow d_k \rightarrow d_j\}$. If we use the same method in Section 5.1.1 to set the weight of $e<d_i, d_j>$, the path that contains $e<d_i, d_j>$ in the CTT can only represent the storage strategy of datasets in the main branch, where the sub-branches are not represented. For example, in Figure 5.3, the path from $d_s$ to $d_8$ that contains the out-block edge $e<d_5, d_8>$, does not represent the storage statuses of datasets $d_3$ and $d_4$, and the length of the path also does not contain the cost rates of $d_3$ and $d_4$, if we use the method in Section 5.1.1 to calculate the weights of the edges. Hence, to maintain the mapping between the paths and the storage strategies, the weights of out-block and over-block edges should contain the minimum cost rates of the datasets in the sub-branches of the block.

Based on the reasons above, if $e<d_i, d_j>$ is an out-block edge or over-block edge, we define its weight as

\[
\omega <d_i, d_j> = y_j + \sum_{|d_k| d_k \in MB \land d_i \rightarrow d_k \rightarrow d_j} \left( genCost(d_k) * v_k \right) + \left( \sum_{|d_k| d_k \in SB} CostR_k \right)_{S'}
\]

In formula (5.2), $\left( \sum_{|d_k| d_k \in SB} CostR_k \right)_{S'}$ is the sum of cost rates of the datasets that are in the sub-branches of the block, where $S'$ is the MCSS of the sub-branches. This formula guarantees that the length of the shortest path with an out-block edge or over-block edge still equals the minimum cost rate of the datasets, which is

\[
P_{\min} <d_s, d_j> = \left( \sum_{|d_k| d_k \in DDG \land d_s \rightarrow d_k \rightarrow d_j} CostR_k \right)_{S'}.
\]

Hence, to calculate the weights of out-block and over-block edges, we have to find the MCSS of the datasets that are in the sub-branches of the block. For example, the weight of the edge $e<d_5, d_8>$ in Figure 5.3 is

\[
\omega <d_5, d_8> = y_8 + genCost(d_8) * v_6 + genCost(d_7) * v_7 + (CostR_3 + CostR_4)_{S'},
\]
where we have to find the MCSS of datasets $d_3$ and $d_4$.

However, for any sub-branch, the MCSS is dependant on the storage status of the datasets preceding and succeeding the block (i.e. stored adjacent predecessor and successor of the sub-branches).

If $e<d_i, d_j>$ is an over-block edge, according to rules of setting weight, $d_i$ and $d_j$ are stored datasets, and the datasets between $d_i$ and $d_j$ in the main branch, \( \{ d_k | d_k \in MB \land d_i \rightarrow d_k \rightarrow d_j \} \), are deleted. Hence, $d_i$ and $d_j$ are the stored adjacent predecessor and successor of the sub-branch. If the rest of datasets within the block form a linear DDG, we can use the linear CTT-SP algorithm introduced in Section 5.1.1 to find its MCSS, where in the first step we have to use $d_i$ and $d_j$ as the start and end datasets. For example, to calculate the weight of over-block edge $e<d_i, d_j>$ in Figure 5.3, we have to find the MCSS $S'$ of sub-branch \{ $d_i, d_j$ \} by the linear CTT-SP algorithm, given, $d_i$ is the start dataset and $d_j$ is the end dataset. Otherwise, if the rest of datasets within the block do not form a linear DDG, we have to recursively call the CTT-SP algorithm to find the MCSS of sub-branches, which will be introduced in Section 5.1.3. Hence, the weight of an over-block edge can be calculated.

If $e<d_i, d_j>$ is an out-block edge, we only know the stored adjacent successor of the sub-branches is $d_j$. However, the MCSS of the sub-branches is also dependant on the stored adjacent predecessor, which is unknown for an out-block edge. Hence, given different stored adjacent predecessors, the weight of an out-block edge would be different. For example, to calculate the weight of out-block edge $e<d_i, d_j>$ in Figure 5.3, we need to find the MCSS $S'$ of the sub-branch \{ $d_i, d_j$ \}, where we only know the stored adjacent successor $d_j$. However, $S'$ may be different depending on the storage statuses of $d_i$ and $d_j$. Hence, we have to create multiple CTTs for the DDG with a block, in order to calculate the weights of out-block edges in different situations, as detailed next.
5.1.2.3 Steps of finding MCSS for DDG with one sub-branch in one block

In this sub-section, we extend the linear CTT-SP algorithm to find its MCSS for DDG with one sub-branch in the block. As discussed in Section 5.1.2.2, depending on different stored preceding datasets of the block, the weight of an out-block edge may be different. Hence multiple CTTs are needed to represent these different situations, and the MCSS is the shortest path among all the CTTs.

To find the MCSS for a DDG with one sub-branch in the block, we need the following two theorems.

**Theorem 5.2:** The selection of main branch in the DDG to construct CTT has no impact on finding the MCSS.

**Theorem 5.3:** The Dijkstra algorithm is still applicable to find the MCSS of the DDG with one block.

Based on these two theorems, we design the algorithm for finding the MCSS for the DDG with one block. The main steps are as follows.

**Step 1:** Construct the initial CTT of the DDG. According to Theorem 5.2, we choose an arbitrary branch in the DDG as the main branch and add cost edges to construct the CTT. In the CTT, for the ordinary edges and in-block edges, we set their weights based on formula (5.1) in Section 5.1.1. For the over-block edges, we set their weights according to formula (5.2) by calling the linear CTT-SP algorithm to find the MCSS of the sub-branch, which is introduced in Section 5.1.2.2. For the out-block edges, we initialise their weights as infinity. The initial CTT is shown in Figure 5.4 (a). We create a CTT set and add the initial CTT to it.

**Step 2:** Based on Theorem 5.3, start the Dijkstra algorithm to find the shortest path from $d_i$ to $d_e$, which applies to all CTTs in the CTT set. We use $F$ to denote the set of datasets discovered by the Dijkstra algorithm. When a new edge $e<d_i, d_j>$ is discovered, we first add $d_j$ to $F$, and then check whether $e<d_i, d_j>$ is an in-block edge.
or not. If not, we continue to find the next edge by the Dijkstra algorithm until \( d_e \) is reached which would terminate the algorithm. If \( e^{d_i, d_j} \) is an in-block edge, create a new CTT (see steps 2.1 - 2.3 next) because whenever an in-block edge is discovered, a stored adjacent predecessor of the sub-branch is identified, and this dataset will be used in calculating the weights of out-block edges. Then we continue the Dijkstra algorithm to find the next edge.

**Step 2.1:** In the case where in-block edge \( e^{d_i, d_j} \) is discovered, based on the current CTT, create CTT(\( e^{d_i, d_j} \)) as shown in Figure 5.4 (b). First, we copy all the information of the current CTT to new CTT(\( e^{d_i, d_j} \)). Second, we update the weights of all the in-block edges in CTT(\( e^{d_i, d_j} \)) as infinity, except \( e^{d_i, d_j} \). This guarantees that dataset \( d_i \) is the stored adjacent predecessor of the sub-branch in all the paths of CTT(\( e^{d_i, d_j} \)). Third, we update the weights of all the out-block edges in CTT(\( e^{d_i, d_j} \)) as described next.

**Step 2.2:** Calculate the weight of an out-block edge \( e^{d_h, d_k} \) in CTT(\( e^{d_i, d_j} \)). As discussed in Section 5.1.2.2, to calculate the weight of \( e^{d_h, d_k} \) according to formula (5.2), we have to find the MCSS of the sub-branch in the block. From Figure 5.4 (b) we can see that the sub-branch is \( \{d'_1, d'_2, \ldots, d'_m\} \), which is a linear DDG. We can find its MCSS by using the linear CTT-SP algorithm described in Section 5.1.1, given that \( d_i \) is the start dataset and \( d_k \) is the end dataset. The CTT created for the sub-branch is depicted in Figure 5.4 (c).

**Step 2.3:** Add new CTT(\( e^{d_i, d_j} \)) to the CTT set.
5.1.3 Minimum Cost Benchmarking Algorithm for General DDG

In the real world applications, the structure of DDG could be complex, i.e. there may exist more than one block in a DDG. However, to find the MCSS of a general DDG, no matter how complex the DDG’s structure is, we can deduce the calculation process to the linear DDG situations by recursively calling the algorithm introduced in Section 5.1.2. In this sub-section we present the general CTT-SP algorithm for benchmarking. First we discuss different situations of the algorithm for a general DDG, and then we give the pseudo-code of finding the MCSS for general DDG.
5.1.3.1 General CTT-SP algorithm for different situations

The complex structure of a DDG can be viewed as a combination of many blocks. Following the algorithm steps introduced in Section 5.1.2.3, we choose an arbitrary branch from the start dataset $d_s$ to the end dataset $d_e$ as the main branch to construct the initial CTT and create multiple CTTs for different in-block edges discovered by the Dijkstra algorithm. In the process of calculating the weights of out-block and over-block edges, there are two new situations for finding the MCSS of the sub-branches.

1) The sub-branches may have more than one stored adjacent predecessor. For example, $e < d_i , d_j >$ in Figure 5.5 is an out-block edge of block $B_1$, and also an in-block edge of block $B_2$. In the algorithm, if edge $e < d_i , d_j >$ is found by the Dijkstra algorithm, we create a new CTT($e < d_i , d_j >$) from the current CTT, since $e < d_i , d_j >$ is an in-block edge of block $B_2$. To calculate the weights of out-block edges in CTT($e < d_i , d_j >$), for example $e < d_i , d_j >$ in Figure 5.5, we need to find the MCSS of the sub-branch $\{d_1', d_2', \ldots , d_m'\}$ of block $B_2$. However, because $e < d_i , d_j >$ is also an out-block edge of $B_1$, $d_i$ is not the only dataset in $d_i$’s provSet. To calculate the generation cost of $d_i'$, we need to find its stored provenance datasets from sub-branch $Br_1$ of block $B_1$.

![Figure 5.5 Sub-branch with more than one stored adjacent predecessor](image)

2) The sub-branches are a general DDG which also has branches. In this situation, we need to recursively call the general CTT-SP algorithm to find its MCSS. For example, $e < d_i , d_j >$ in Figure 5.6 is an in-block edge of blocks $B_1$ and $B_2$. If $e < d_i , d_j >$ is selected by the algorithm, we need to create a new CTT($e < d_i , d_j >$). To calculate the weight of $e < d_i , d_j >$ in Figure 5.6, which is an out-block edge of both $B_1$ and $B_2$, we need to find the MCSS of the sub-branches $Br_1$ and $Br_2$. Hence we have
to recursively call the general CTT-SP algorithm for the DDG \( Br_1 \cup Br_2 \), given the start dataset \( d_i \) and the end dataset \( d_k \).

Figure 5.6 Sub-branch with branches

Hence, given a general DDG, its structure can be viewed as a combination of many blocks. By recursively calling the general CTT-SP algorithm for the sub-branches, we can eventually find the MCSS of the whole DDG. Figure 5.7 shows an example of general DDG. To create CTT\((e<d_i, d_j>)\), we need to calculate the weights of all the out-block edges. For example, for an out-block edge \( e<d_h, d_k> \), we need to further find the MCSS of the sub-branches \( \{d_u \mid d_u \in DDG \land d_u \rightarrow d_k \land d_u \leftrightarrow d_j \land d_u \leftrightarrow d_h\} \), as shadowed in Figure 5.7, given the start dataset \( d_i \) and the end dataset \( d_k \).

Figure 5.7 CTT for general DDG
5.1.3.2 Pseudo-code of general CTT-SP algorithm

Figure 5.8 shows the pseudo-code of the general CTT-SP algorithm. At the beginning, we choose an arbitrary branch from $d_s$ to $d_e$ as the main branch to construct the initial CTT (lines 1-21), where we need to recursively call the general CTT-SP algorithm in calculating the weights for over-block edges (lines 11-14). Then we start the Dijkstra algorithm (lines 22-50). Whenever an in-block edge is found, we construct a new CTT with the following steps. First, we create a copy of the current CTT, in which the in-block edge is found (line 31). Next, we update the weights of edges: lines 32 to 34 are for updating the weights of in-block edges and lines 35 to 49 are for updating the weights of out-block edges. If the sub-branch is a linear DDG, we call the linear CTT-SP algorithm described in Figure 5.2, otherwise we recursively call the general CTT-SP algorithm (lines 39-42). At last, we add the new CTT to the CTTSet (line 50) and continue the Dijkstra algorithm to find the next edge. When the end dataset $d_e$ is reached, the algorithm ends with the MCSS and the minimum cost benchmark returned (lines 51-53).

From the pseudo-code in Figure 5.8, we can see that recursive calls (line 14 and line 42) exist in the general CTT-SP algorithm, which makes the algorithm’s complexity highly dependant on the structure of DDG. Next, we analyse the worst case scenario of the algorithm and show that the time complexity is polynomial.

In Figure 5.8, pseudo-code lines 1 to 21 are for constructing one CTT, i.e. initial CTT. From pseudo-code lines 24 to 50 of the general CTT-SP algorithm, many CTTs are created for the DDG during the deployment of the Dijkstra algorithm, which determine the algorithm’s computation complexity. The maximum number of the created CTTs is smaller than the number of datasets in the main branch, which is in the magnitude of $n$. Hence, if we denote the time complexity of the general CTT-SP algorithm as $F_r(n)$, we have the recursive equation as follows:

$$
\begin{cases}
F_0(n) = O(n^4) \\
F_r(n) = n^3 \left( F_{r-1}(n_{r-1}) + n^2 \right), \quad r > 0
\end{cases}
$$

(5.3)
Algorithm: General_CTT-SP

Input: start dataset $d_s$; end dataset $d_e$; a general DDG; //Include $d_s$ and $d_e$

Output: $S, SCR$; //MCSS of the DDG and the minimum cost benchmark

01. Get a main branch $MB$ from DDG;
02. for (every dataset $d_i$ in $MB$) //Create initial CTT
03. for (every dataset $d_j$, where $d_j \in MB \land d_i \rightarrow d_j$) //Create an edge
04. Create $e < d_i, d_j >$
05. if ($\exists d_k \in DDG \land d_i \Leftrightarrow d_k \land d_k \rightarrow d_j$) //Is $e$ is an out-block edge
06. Set $\omega < d_i, d_j > = \infty$
07. else //Calculate the weight of the edge
08. \hspace{1cm} weight = 0;
09. if ($\exists d_i \notin MB \land d_i \rightarrow d_k \rightarrow d_j$) //Is $e$ is an over-block edge
10. \hspace{1cm} $SB = \{d_i \mid d_i \notin MB \land d_i \rightarrow d_k \rightarrow d_j\}$; //Get the sub-branches $SB$
11. if ($SB$ is linear) //Find the minimum cost storage strategy of $SB$
12. \hspace{1cm} $S' = \text{Linear_CTT-SP}(d_i, d_j, SB)$;
13. else
14. \hspace{1cm} $S' = \text{General_CTT-SP}(d_i, d_j, SB)$;
15. weight = weight + $\sum_{d_i \in SB} \text{CostR}$
16. for (every dataset $d_o$, where $d_o \in MB \land d_o \rightarrow d_j$) //Datasets in main branch
17. \hspace{1cm} genCost = 0;
18. for (every dataset $d_o$, where $d_o \in MB \land d_o \rightarrow d_j$)
19. \hspace{1cm} genCost = genCost + $x_o$;
20. \hspace{1cm} weight = weight + $(x_o + \text{genCost}) \cdot y_o$; //Sum of generation cost rates
21. \hspace{1cm} Set $\omega < d_o, d_j > = weight + y_o$; //Set weight to the edge
22. $\text{CTTSet} = \{\text{CTT}_w\}$; //Set of all the created CTTs
23. $F = \emptyset$; //Set of datasets discovered by Dijkstra algorithm
24. while ($d_i$ is not in $F$) //Find the next edge for the shortest path
25. for (every CTT in $\text{CTTSet}$) //Find the next edge by Dijkstra algorithm;
26. Get the current shortest path in all the CTTs, which is with the edge $e < d_i, d_j > \in \text{CTT}'$
27. Add $d_j$ to $F$;
28. if ($\exists d_k \in DDG \land d_i \rightarrow d_k \land d_k \Leftrightarrow d_j$) //Is $e$ is an in-block edge
29. \hspace{1cm} $BSet = \{B_p \mid B_p \in DDG \land d_i \in B_p \land d_j \in B_p\}$; //Blocks that contains $d_i$ but not $d_j$
30. Create a copy of CTT' denoted as CTT($e < d_i, d_j >$); //Create a new CTT
31. for (every $B_p \in BSet$) //Update the weights of the in-block edges
32. \hspace{1cm} $SB = \{d_p \mid d_p \in DDG \land d_i \rightarrow d_p \rightarrow d_k \land d_k \Leftrightarrow d_j \land d_j \Leftrightarrow d_i\}$; //Get the sub-branches $SB$
33. if ($SB$ is linear) //Find the minimum cost storage strategy of $SB$
34. \hspace{1cm} $S' = \text{Linear_CTT-SP}(d_i, d_j, SB)$;
35. else
36. \hspace{1cm} $S' = \text{General_CTT-SP}(d_i, d_j, SB)$;
37. weight = $\sum_{d_i \in SB} \text{CostR}$
38. for (every dataset $d_o$, where $d_o \in MB \land d_o \rightarrow d_j$) //Datasets in main branch
39. \hspace{1cm} genCost = 0;
40. for (every dataset $d_o$, where $d_o \in MB \land d_o \rightarrow d_j$)
41. \hspace{1cm} genCost = genCost + $x_o$;
42. \hspace{1cm} weight = weight + $(x_o + \text{genCost}) \cdot y_o$; //Sum of generation cost rates
43. \hspace{1cm} Set $\omega < d_o, d_j > = weight + y_o$; //Set weight to the out-block edge
44. Add CTT($e < d_i, d_j >$) to $\text{CTTSet}$;
45. $S = \text{set of datasets that the shortest path from } d_s \text{ to } d_e \text{ has traversed}$;
46. $SCR = \sum_{d_i \in \text{DDG}} \text{CostR}$;
47. Return $S, SCR$;

Figure 5.8 Pseudo-code of general CTT-SP algorithm for benchmarking
In equation (5.3), \( n \) is the number of datasets in the DDG, \( n_{(r-1)} \) is the number of datasets in the sub-branches, and \( r \) is the maximum level of the recursive calls, especially \( F_{\delta}(n) \) denotes the situation of linear DDG, where the linear CTT-SP algorithm needs to be called (i.e. pseudo-code in Figure 5.2).

Intuitively, in equation (5.3), \( F_{r}(n) \) seems to have an exponential complexity (i.e. NP-hard) depending on the level of recursive calls. However, in our scenario, \( F_{r}(n) \) is polynomial because the recursive call is to find the MCSS of given sub-branches in DDG which has a limited solution space. Hence, we can use the iterative method [64] to solve the recursive equation and derive the computation complexity of the general CTT-SP algorithm.

If we assume that we have already found the MCSSs for all sub-branches which means without taking the impact of recursive calls into account, the general CTT-SP algorithm has a time complexity of \( O(n^5) \), because there are five nested loops in the pseudo-code in Figure 5.8 (lines 24, 35, 36, 44, 46). Formally, we can transform equation (5.3) to the following:

\[
F_{r}(n) = n^3 \times \left( O(1) + n^2 \right) + f_{rec} \left( F_{r-1}(n_{(r-1)}) \right) \\
= O(n^5) + f_{rec} \left( F_{r-1}(n_{(r-1)}) \right)
\]

(5.4)

In equation (5.4), function \( f_{rec} \) denotes the complexity of recursive calls, i.e. calculating the minimum cost storage strategies of all sub-branches. Next, we analyse the complexity of recursive calls.

For a sub-branch of a general DDG, given different start dataset and end dataset, its MCSS may be different. Figure 5.9 shows a sub-branch of DDG with \( w \) datasets. We assume \( d_1 \)'s direct predecessors and \( d_w \)'s direct successors are all stored, then we can find a MCSS of the sub-branch. We denote the first stored dataset as \( d_u \) and the last stored dataset as \( d_v \) in the strategy, which is shown in Figure 5.9. If \( d_1 \)'s adjacent stored predecessors are changed, the MCSS may be different as well. Because the generation cost of \( d_1 \) is larger than storing the direct predecessors, the first stored dataset in the new strategy must be one of the datasets from \( d_1 \) to \( d_u \). Similarly, if \( d_w \)'s adjacent stored successors are changed, the last stored dataset in the
new strategy must be one of the datasets from \( d_v \) to \( d_w \). Hence, given different start and end datasets, a sub-branch of DDG has at most \( u^*(w-v) \) different minimum cost storage strategies, which are in the magnitude of \( w^2 \). Hence, we have the conclusion that for any sub-branches of DDG with \( w \) datasets, there are at most \( w^2 \) different minimum cost storage strategies, given different start and end datasets. Hence, given any sub-branches in DDG at any level of recursive calls, say level \( h \), we have the time complexity \( F_h(w)\times w^2 \) for finding all the possible minimum cost storage strategies.

A sub-branch in DDG

![A sub-branch in DDG](image)

**Figure 5.9 A sub-branch in DDG**

If we assume that there are \( m \) different sub-branches of recursive calls at level \( h \) that we have to find their minimum cost storage strategies, we have the complexity of recursive calls at this level as follows:

\[
 f_{rec}(F_h(n_h)) \leq \sum_{i=1}^{m} \left(F_{h,i}(n_{h,i}) \times n_{h,i}^2 \right) 
\]  
(5.5)

With formula (5.5), we can further transform equation (5.4) and iteratively derive the time complexity of the general CTT-SP algorithm.

Therefore, the entire iteration process from equation (5.3) is shown as follows:

\[
 F_r(n) = n^3 \times \left(F_{r-1}(n_{r-1}) + n^2 \right) \\
= O(n^5) + f_{rec}\left(F_{r-1}(n_{r-1})\right) \quad // \text{from equation(5.4)} \\
\leq O(n^5) + \sum_{i=1}^{m} \left(F_{r-1}(n_{(r-1)i}) \times n_{(r-1)i}^2 \right) \quad // \text{from formula(5.5)} \\
= O(n^5) + \sum_{i=1}^{m} \left(n_{(r-1)i}^3 \times \left(F_{r-2}(n_{(r-2)i}) + n_{(r-1)i}^2 \right) \times n_{(r-1)i}^2 \right) \quad // \text{recursion}
\]
\[ \leq O(n^5) + \sum_{i=1}^{m-1} \left( O \left( (n_{(r-1),i})^5 \right) \cdot n_{(r-1),i}^2 \right) \]

\[ + \sum_{i=1}^{m-2} \left( F_{r-2} (n_{(r-2),i}) \cdot n_{(r-2),i}^2 \right) \quad // from \text{ equation}(5.4) \& \text{ formula}(5.5) \]

\[ \leq O(n^5) + \sum_{i=1}^{m-2} \left( O \left( (n_{(r-1),i})^5 \right) \cdot n_{(r-1),i}^2 \right) + \ldots + \sum_{i=1}^{m_0} \left( F_0 (n_{0,i}) \cdot n_{0,i}^2 \right) \quad // \text{iteration} \]

\[ = O(n^5) + \sum_{j=r-1}^{1} \left( \sum_{i=1}^{m_j} O \left( (n_{j,i})^5 \right) \cdot n_{j,i}^2 \right) + \sum_{i=1}^{m_0} \left( O(n_{0,i}^4) \cdot n_{0,i}^2 \right) \quad // F_0 (n) = O(n^4) \]

\[ \leq r \cdot m \cdot O(n^5) \cdot n^2 \quad // m = \max_{i=0}^{j} (m_i) \]

\[ \leq O(n^9) \quad // r < n, \quad m < n \]

Hence, the worst case time complexity of the general CTT-SP algorithm is $O(n^9)$.

Based on the complexity analysis, we can see that the general CTT-SP algorithm provides a benchmarking approach for a seemingly NP-hard problem with a polynomial solution.

In Chapter 7, we will use experiment results to further demonstrate this on-demand benchmarking approach.

### 5.2 Dynamic on-the-fly Minimum Cost Benchmarking Approach

In this section, we describe our novel on-the-fly minimum cost benchmarking approach in detail. The basic idea is that we divide the whole DDG into smaller linear DDG segments (DDG_LS) and create a Partitioned Solution Space (PSS) for every segment. PSS saves all the possible MCSSs of the DDG segment, which are calculated by the CTT-SP algorithm. The minimum cost benchmark of the whole DDG can be calculated by merging the PSSs. Whenever new datasets are generated and/or existing datasets’ usage frequencies are changed, the new benchmark can be
dynamically located on the fly from the pre-calculated PSSs with only calling the CTT-SP algorithm on the small local DDG segment for adjustment. Hence we can keep the minimum cost benchmark updated on the fly so that users’ benchmarking requests can be instantly responded.

5.2.1 PSS for a DDG_LS

PSS is the basis of our dynamic benchmarking approach. In this sub-section, we first explain the reason why there exists a solution space of MCSSs for a DDG_LS. Then we introduce some properties of the solution space and further investigate how the MCSSs are distributed in a PSS.

5.2.1.1 Different MCSSs of a DDG_LS in a solution space

Generally speaking, a DDG_LS would only have one MCSS for storing the datasets in it. However, due to different preceding and succeeding datasets’ storage statuses, there would be different corresponding MCSSs, one for each status.

The CTT-SP algorithm can be utilised on not only independent DDGs but also DDG_LSs, where the difference is the selection of start and end datasets for constructing the CTT. For an independent DDG, we add two virtual datasets $d_s$ and $d_e$ as start and end datasets to construct the CTT as shown in Figure 5.1. However, for the CTT of a DDG_LS, the start dataset $d_s$ is the nearest stored preceding dataset to the DDG_LS, and the end dataset $d_e$ is the nearest stored succeeding dataset to the DDG_LS. Figure 5.10 shows an example of CTT for a DDG_LS.

![Figure 5.10 CTT for a DDG_LS](image-url)
Hence, given different start and end datasets, the MCSS of a DDG_LS may be different. This is because 1) the deleted preceding datasets impact on the generation cost of datasets in the DDG_LS; 2) the generation of the deleted succeeding datasets need to use datasets in the DDG_LS.

Next, we analyse how the preceding and succeeding datasets of the DDG_LS impact on its MCSS.

**Theorem 5.4:** For a DDG_LS, only the generation cost of its deleted preceding datasets and the usage frequencies of its deleted succeeding datasets impact on its MCSS.

Based on Theorem 5.4, for a DDG_LS \{d_1, d_2, … d_n\}, we introduce two definitions:

- \( X = \sum_{d_i \in DDG \land d_i \rightarrow d_i \rightarrow d_i} \) is the sum of preceding datasets generation costs of a DDG_LS, where \( d_i \) is a deleted preceding dataset.

- \( V = \sum_{d_j \in DDG \land d_j \rightarrow d_j \rightarrow d_j} \) is the sum of succeeding datasets usage frequencies of a DDG_LS, where \( d_j \) is a deleted succeeding dataset.

For different start and end datasets, the values of \( X \) and \( V \) are different, and the MCSS of the DDG_LS may also be different. In other words, given different \( X \) and \( V \), there exist different MCSSs for storing the DDG_LS. We denote an MCSS as \( S_{i,j} \), where \( d_i \) and \( d_j \) are the first and last stored datasets in the strategy, which could be any datasets in the DDG_LS. Conversely, any two datasets \( d_i \) and \( d_j \) in the DDG_LS may be the first and last stored datasets of an MCSS. Hence, theoretically, the number of different MCSSs for a DDG_LS is in the magnitude of \( n_i^2 \), where \( n_i \) is the number of datasets in the DDG_LS.

### 5.2.1.2 Range of MCSSs’ cost rates for a DDG_LS

Different MCSSs have different cost rates (i.e. \( SCR \) defined in formula (4.4) in Section 4.3) for storing the DDG_LS. Because DDG_LS is a segment of the whole DDG, the total cost rate of storing it includes not only the cost rate of itself, but also
the cost rate of generating the deleted preceding and succeeding datasets. Hence, given any \(X\) and \(V\), and the corresponding MCSS \(S_{i,j}\), we denote the total cost rate of storing the DDG_LS \(\{d_1, d_2, \ldots d_{nl}\}\) as \(TCR_{i,j}\), where

\[
TCR_{i,j} = X \sum_{k=1}^{i-1} V_k + SCR_{i,j} + V \sum_{k=j+1}^{n_l} x_k
\]  

(5.6)

In formula (5.6), \(SCR_{i,j}\) is the cost rate of storing the DDG_LS with the storage strategy \(S_{i,j}\), assuming that the direct preceding and succeeding datasets of DDG_LS are stored. Formally,

\[
SCR_{i,j} = \left\{ \sum_{d_k \in DDG_LS} CostR_k \right\}_{S_{i,j}}
\]  

(5.7)

An important difference between \(TCR_{i,j}\) and \(SCR_{i,j}\) is that \(TCR_{i,j}\) is a variable for a storage strategy depending on the value of \(X\) and \(V\) (see formula (5.6)), whereas \(SCR_{i,j}\) is a constant for a specific storage strategy (see formula (5.7)).

For a DDG_LS, one extreme situation of \((X=0, V=0)\) means that the start and end datasets are the direct preceding and succeeding datasets of the DDG_LS. Hence we can deem the DDG_LS as an independent DDG and directly call the CTT-SP algorithm to find its MCSS. In this situation, MCSS \(S_{u,v}\) found is the minimum \(SCR_{u,v}\) for storing the DDG_LS among other MCSSs, where \(TCR_{u,v} = SCR_{u,v}\). We denote \(S_{u,v}\) as \(S_{min}\) and \(SCR_{u,v}\) as \(SCR_{min}\).

The other extreme situation is that the start and end datasets are very far from the current DDG_LS, i.e. \(X>y_1/v_1, V>y_{nl}/x_{nl}\). Obviously, in this situation the first dataset \(d_1\) and the last dataset \(d_{nl}\) in the DDG_LS should be stored. Hence we can deem \(d_1\) and \(d_{nl}\) as the start and end datasets and call the CTT-SP algorithm for the datasets between \(d_1\) and \(d_{nl}\). The found strategy together with \(d_1\) and \(d_{nl}\) form the MCSS of the DDG_LS in this situation denoted as \(S_{l, nl}\), where we also have \(TCR_{l, nl} = SCR_{l, nl}\). We denote \(S_{l, nl}\) as \(S_{max}\) and \(SCR_{l, nl}\) as \(SCR_{max}\).

**Theorem 5.5:** Given a DDG_LS \(\{d_1, d_2, \ldots d_{nl}\}\), \(SCR_{min}\) is the cost rate of MCSS \(S_{u,v}\) with \(X=0, V=0\), and \(SCR_{max}\) is the cost rate of MCSS \(S_{l, nl}\) with \(X>y_1/v_1, V>y_{nl}/x_{nl}\). Then we have \(SCR_{min} < SCR_{i,j} < SCR_{max}\), where \(SCR_{i,j}\) is the cost rate.
of MCSS $S_{i,j}$ with any given $X$ and $V$.

\[
X = \sum_{i} x_i \\
V = \sum_{j} v_j
\]

**Figure 5.11 Different MCSSs for a DDG_LS**

Figure 5.11 shows the MCSSs for a DDG_LS whose $SCR$ values are in the valid range indicated in Theorem 5.5. We can further find all these strategies and save them in a strategy set, denoted as $S_{All}$. Figure 5.12 shows the pseudo code of finding $S_{All}$. The essence of this algorithm is the utilisation of the CTT-SP algorithm. Given a DDG_LS $\{d_1, d_2, \ldots, d_n\}$, we first create the CTT for it (line 1). Then (line 2), we call the Dijkstra algorithm on the CTT to find the shortest path from $d_s$ to $d_e$ which are the two virtual datasets added when creating the CTT. The corresponding MCSS $S_{u,v}$ is $S_{min}$ with $SCR_{min}$, where $d_u$ and $d_v$ are the first and last stored datasets in this MCSS. Similarly, we find $S_{max}$ with $SCR_{max}$ (line 3). Next, we initialise $S_{All}$ and $S_{max}$ (lines 4-5) and go through all the possible positions of the first and last stored datasets and find the corresponding MCSSs (lines 6-9). We eliminate the MCSSs with invalid SCR values according to Theorem 5.5 (line 10) and save the valid MCSSs in $S_{All}$ (line 11).

The time complexity of creating the CTT is $O(n_i^4)$ (line 1) according to the CTT-SP algorithm [91], where $n_i$ is the number of datasets in the DDG_LS. Next, the time complexity of finding all the possible MCSSs is $n_i^2$ (as indicated earlier at the end of Section 5.2.1.1) (lines 6-7) multiplying the time complexity of the Dijkstra algorithm, which is $O(n_i^2)$ (line 9). Hence the total time complexity of finding $S_{All}$ is $O(n_i^4)$. 

---

\[
S_{max}(X>y_{v_1}, V>y_{x_1})
\]
Figure 5.12 Pseudo code of finding S_All

As discussed above, given any X and V, there exists one MCSS for storing the DDG_LS in the set of S_All. Hence we create a coordinate of X and V to represent the solution space of all possible MCSSs for a DDG_LS. Furthermore we can calculate the distribution of the MCSSs in the solution space and call it PSS as described next.

5.2.1.3 Distribution of MCSSs in the PSS of a DDG_LS

We start with analysing the relationship of two MCSSs in the solution space. We assume that \( S_{ij} \) and \( S_{i'j'} \) be two MCSSs in S_All of a DDG_LS \( \{d_1, d_2, \ldots d_n\} \) and \( SCR_{ij} < SCR_{i'j'} \). The border of \( S_{ij} \) and \( S_{i'j'} \) in the solution space is that given particular X and V, the total cost rates (TCR) of storing the DDG_LS with \( S_{ij} \) and \( S_{i'j'} \) are equal. Hence we have

\[
TCR_{i,j} = TCR_{i',j'}
\]

\[
\Rightarrow X \sum_{k=1}^{i-1} v_k + SCR_{i,j} + V \sum_{k=j+1}^{n} x_k = X \sum_{k=1}^{i'-1} v_k + SCR_{i',j'} + V \sum_{k=j'+1}^{n} x_k
\]

\[
\Rightarrow (\sum_{k=1}^{i-1} v_k - \sum_{k=1}^{i'-1} v_k) * X + (\sum_{k=j+1}^{n} x_k - \sum_{k=j'+1}^{n} x_k) * V + (SCR_{i',j'} - SCR_{i,j}) = 0 \quad (5.8)
\]
From this equation we can see that the border of \( S_{i,j} \) and \( S_{i',j'} \) in the solution space is a straight line. Given different relationships of \( d_i \) and \( d_i' \), \( d_j \) and \( d_j' \), there are four different situations.

1) \( d_i \rightarrow d_i \land d_j \rightarrow d_j' \), as shown in Figure 5.13 (a), formula (5.8) can be further simplified to:

\[
\left( \sum_{k=j}^{i-1} v_k \right) \ast X + \left( \sum_{k=j+1}^{i} x_k \right) \ast V - (SCR_{i,j} - SCR_{i,j'}) = 0
\]

(5.9)

2) \( d_i \rightarrow d_i \land d_j \rightarrow d_j' \), as shown in Figure 5.13 (b), formula (5.8) can be further simplified to:

\[
\left( \sum_{k=j}^{i-1} v_k \right) \ast X - \left( \sum_{k=j+1}^{i} x_k \right) \ast V + (SCR_{i,j} - SCR_{i,j'}) = 0
\]

(5.10)

3) \( d_i \rightarrow d_i \land d_j \rightarrow d_j' \), as shown in Figure 5.13 (c), formula (5.8) can be further simplified to:

\[
\left( \sum_{k=j}^{i-1} v_k \right) \ast X - \left( \sum_{k=j+1}^{i} x_k \right) \ast V - (SCR_{i,j} - SCR_{i,j'}) = 0
\]

(5.11)

4) \( d_i \rightarrow d_i \land d_j \rightarrow d_j' \), as shown in Figure 5.13 (d), formula (5.8) can be further simplified to:

\[
\left( \sum_{k=j}^{i-1} v_k \right) \ast X + \left( \sum_{k=j+1}^{i} x_k \right) \ast V + (SCR_{i,j} - SCR_{i,j'}) = 0
\]

Because \( X \) and \( V \) are positive values, \( S_{i',j'} \) can never be an eligible MCSS for the DDG_LS in the situation of Figure 5.13 (d). Hence we have a property of the MCSSs of a DDG_LS as follows:

\[
(S_{i,j}, S_{i',j'} \in S_{\text{All}}) \land (SCR_{i,j} \leq SCR_{i',j'}) \land (d_i \rightarrow d_i \land d_j \rightarrow d_j') \implies S_{i',j'} \notin PSS
\]

(5.12)
Figure 5.13 Examples of partition lines in a solution space

Hence, for any two MCSSs, we can find the partition line in the solution space which is one of the three formulas listed above, namely formulas (5.9), (5.10) or (5.11). According to property (5.12), we can further eliminate some MCSSs from S_All, which should not be in the solution space. We refer the eligible MCSSs in S_All as S_ini that is the initial input for calculating the solution space. From Figure
5.14, we can see that the time complexity of eliminating $S_{All}$ is $O(n_s^2)$, where $n_s$ is the number of MCSSs in $S_{All}$.

<table>
<thead>
<tr>
<th>Algorithm: Eliminate $S_{All}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: $S_{All}$</td>
</tr>
<tr>
<td>Output: $S_{ini}$</td>
</tr>
</tbody>
</table>

01. for ( every $S_{i,j}\in S_{All}$ )
02. for ( every $S_{i',j'}\in S_{All} \land SCR_{i,j} \leq SCR_{i',j'}$ )
03. if ($d_i\rightarrow d'_i \land d_j\rightarrow d'_j$)
04. Eliminate $S_{i',j'}$ from $S_{All}$;
05. Return the eliminated $S_{All}$ as $S_{ini}$;

Figure 5.14 Pseudo code of eliminating $S_{All}$

From the above discussion, we can see that the solution space of a DDG_LS is partitioned by lines into different areas, which forms the PSS. In the PSS, every area represents an MCSS and the partition lines are the borders. Next we describe our algorithms that can precisely calculate the PSS.

### 5.2.2 Algorithms for Calculating PSS of a DDG_LS

In a solution space, the MCSS of a DDG_LS changes from $S_{min}$ to $S_{max}$ as long as $X$ and $V$ increase. Given MCSS set $S_{ini}$, we calculate the partition line of every two adjacent strategies from $S_{min}$ to $S_{max}$, and gradually partition the solution space. Finally, we derive a PSS, which includes all the possible MCSSs of the DDG_LS. In order to calculate the PSS for a DDG_LS, we need to introduce the following lemma.

**Lemma 5.1:** In the PSS of a DDG_LS, for three MCSSs, if any two of them are adjacent with each other, then the three partition lines between every two MCSSs intersect at one point.

In the statement of Lemma 5.1, two MCSSs are adjacent meaning that the corresponding areas of the two MCSSs in the PSS are adjacent. Figure 5.15 shows an example of Lemma 5.1. We assume that $S_{i,i'}$, $S_{j,j'}$ and $S_{k,k'}$ be three MCSSs, where $SCR_{i,i'} \leq SCR_{j,j'} < SCR_{k,k'}$ as shown in Figure 5.15 (a) and any two of $S_{i,i'}$, $S_{j,j'}$, $S_{k,k'}$ are adjacent as shown in Figure 5.15 (b). Based on the positions of first and last stored datasets, we calculate the three partition lines as follows:
According to Lemma 5.1, these three lines intersect at one point in the PSS as demonstrated in Figure 5.15 (b).

**A DDG_LS**

![Diagram of A DDG_LS](image)

**PSS**

![Diagram of Partition lines of the MCSSs](image)

**Figure 5.15 Example of Lemma 5.1**

Based on Lemma 5.1, we design the algorithm to calculate the PSS for a DDG_LS. The main steps in the pseudo code of this algorithm are shown in Figure 5.16.

As shown in Figure 5.16, the algorithm input is $S_{ini}$, which contains the possible MCSSs of a DDG_LS, and the output is the DDG_LS’s PSS, which is a set of partition lines with start and end points in the solution space. The basic idea of the algorithm is to add the MCSSs to the PSS one by one from $S_{min}$ to $S_{max}$, which contains three main steps:

**Step 1**: initialisation and preparation (lines 1-4). First, we order the MCSSs in $S_{ini}$ by their SCRs and save them in an ascending array list $[S_{min}, S_I \ldots S_{max}]$. Then we calculate the first partition line $L<S_{min}, S_I>$ and its intercepts with the $X$ and $V$
axes, denoted as $X_j$ and $V_j$. Next, we create two ordered array lists $X[]$ and $V[]$ to store the intercepts of the partition lines with the $X$ and $V$ axes respectively. When we add an MCSS to the PSS, $X[]$ and $V[]$ are used to find the first MCSS in the PSS that we start calculating the partition lines. Last, we initialise the PSS with $X$ and $V$ axes, and add $L_{S_{min}, S_j}$ to it.

**Step 2**: calculation of partition lines for an MCSS (lines 5-20). In this step, we start adding the MCSSs (i.e. $[S_{min}, S_1 \ldots S_{max}]$) to the PSS one by one (line 5). To add MCSS $S_i$ to the PSS, first we need to find an adjacent MCSS to it in the PSS, based on which we start calculating the partition lines. To find an adjacent MCSS to $S_i$, we only need to calculate partition line $L_{S_{min}, S_i}$ and insert intercepts $X_i$ and $V_i$ to $X[]$ and $V[]$ (lines 6-7). Adjacent MCSS $S'$ is the corresponding MCSS of the first intercept that is smaller than $X_i$ in $X[]$ or $V_i$ in $V[]$. Next, we add $S_i$ to the PSS and start with calculating the partition line of $S_i$ and $S'$ (line 10). As $S'$ is an existing MCSS in the PSS which represents some areas in the solution space, partition line $L_{S', S_i}$ intersects with the border of $S'$ and new MCSS $S_i$ partially overlaps with existing MCSS $S'$. Hence, we find the borderlines of $S'$ (line 11) and calculate the intersections of $L_{S', S_i}$ (lines 12-13). We also need to save the intersections (lines 14-17), where 1) set $av\_point$ saves all the intersections that will be used in the next step; 2) stack $v\_point$ saves the intersections, which indicate the next MCSS that $S_i$ partitions. Next, we add partition line $L_{S', S_i}$ as well as the endpoints (i.e. the intersections just calculated) to the PSS (line 18), and then, by popping an intersection from $v\_point$ (line 19), we find the next MCSS to partition with $S_i$ which is also the third partition line to that intersection according to Lemma 5.1. This process continues until stack $v\_point$ is empty (line 20) which also means that we have calculated all the partition lines of $S_i$ with its adjacent MCSSs.

**Step 3**: update of the PSS (lines 21-27). After we add a new MCSS into the PSS, some of the old MCSSs may be overlapped. We need to update the existing partition lines in the PSS. As all the intersections of the new joint MCSS’s partition lines are saved in $av\_point$ in step 2, we only need to go through $av\_point$ and update the partition lines’ endpoints (lines 21-25). To update the endpoints, first we need to find which endpoint of the partition line is overlapped by the new joint MCSS. The $valid\_Endpoint$ function (the pseudo code also shown in Figure 5.16) is called to find
Algorithm: Calculate PSS
Input: \( S_{ini} \) //the MCSSs set
Output: \( PSS \) //with partition lines

01. Order \( S_{ini} \) by SCRs and get \( S_{ini}[] = [S_{min}, S_1, \ldots S_{max}] \); //Step 1
02. Calculate \( L<S_{min}, S_i> \), intercept \( X_i, V_i \);
03. Insert \( X_i \) to \( X[] \), \( V_i \) to \( V[] \);
04. Add \( L<S_{min}, S_i>, X\_axis, V\_axis \) to \( PSS \);
05. for (every \( S_i \) in \( S_{ini} \)) //Step 2
06. Calculate \( L<S_{min}, S_i> \),
07. Insert \( X_i \) to \( X[], V_i \) to \( V[] \), find \( S' \);
08. Stack \( v\_points = \Phi \), Set \( av\_points = \Phi \);
09. do
10. Calculate \( L<S', S_i> \);
11. Find \( S':LSet = \{ L<S_{x,v}, S_i> | S_x=S' or S_v=S' \} \);
12. for (every \( L<S_{x,v}, S_i> \) in \( S':LSet \))
13. if ((\( x,v \)) is valid)
14. Add \( \{ L<S_{x,v}, S_i>,(x,v) \} \) to \( av\_points \);
15. if ((\( x,v \)) is on \( X \) or \( V \) axis)
16. Push \( \{ L<S_{x,v}, S_i>,(x,v) \} \) to \( v\_points \);
17. Add \( L<S', S_i> \) to \( PSS \) with the endpoints;
18. \( S' = \) Get MCSS by poping \( v\_point \);
19. while (\( S' = \Phi \))
20. while (every element \( L_1, L_2, (x,v) \) in \( av\_points \)) //Step 3
21. \( (x',v') = \) validEndpoint\((L_1, L_2)\);
22. Update \( L_1 \) with the endpoints \( (x',v') \) and \( (x,v) \);
23. if (\( L_1 \) is an axis)
24. create \( L_{new} \) in \( PSS \) with endpoints \( (x,v) \) and \( \infty \);
25. while (there exist intersections with less than 3 lines)
26. delete the lines;
27. 28. Return \( PSS \)

Function: validEndpoint
Input: \( L_1 \{ (x_1,v_1), (x_2,v_2) \} \) //two endpoints of \( L_1 \)
\( L_2 : A_2 x + B_2 v + C_2 = 0 \) //equation of \( L_2 \)
Output: \( (x, v) \) //the valid endpoint of \( L_1 \)

01. if (\( x_1 = \infty \) \| \( v_2 = \infty \)) Return \( (x_2,v_2) \);
02. else if (\( x_2 = \infty \) \| \( v_2 = \infty \)) Return \( (x_1, v_1) \);
03. \( V\_L_2 = A_2 x_1 + B_2 v_1 + C_2 \);
04. if (\( L_2 \) is type 1 \| \( L_2 \) is type 3)
05. if (\( V\_L_2 < 0 \)) Return \( (x_1, v_1) \);
06. else Return \( (x_2, v_2) \);
07. else if (\( V\_L_2 < 0 \)) Return \( (x_2, v_2) \);
08. else Return \( (x_1, v_1) \);

Figure 5.16 Pseudo code of calculating PSS

the valid endpoint that should be kept in the PSS (line 22). Then, we can update the
partition line by replacing the overlapped endpoint with the new intersection (line 23).
Especially, to update the partition line on the \( X \) or \( V \) axis, we need to create a new
line from the intersection to infinity because the axes cannot be overlapped (lines 24-
25). After updating all the partition lines with the new intersections, we need to check
all the intersections in the PSS. We delete the intersections and the corresponding partition lines that do not conform to Lemma 5.1 (lines 26-27). This is to eliminate the MCSSs that are totally overlapped by the new joint MCSS.

From the pseudo code in Figure 5.16, we can see that the time complexity of the algorithm is \(O(n_s^2n_b)\) (lines 5-20), where \(n_s\) is the number of MCSSs in the PSS, and \(n_b\) is the number of a MCSS’s adjacent MCSSs. Obviously, \(n_b\) is smaller than \(n_s\), hence the time complexity of calculating PSS of a DDG_LS is \(O(n_s^3)\).

![Diagram of PSS](image)

**Figure 5.17 Example of a PSS**

Figure 5.17 illustrates an example of the PSS found by the algorithm in Figure 5.16. With the PSS, given any \(X\) and \(V\), we can locate the corresponding MCSS with time complexity of \(O(n_s)\), where classic algorithms can be found in analytic geometry [76], hence we do not give detailed introduction in this thesis. Furthermore, \(n_s\), the number of MCSSs in the PSS, is usually very small which we will demonstrate in Chapter 7 by experiment results.

### 5.2.3 PSS for a General DDG (or DDG Segment)

The PSS for DDG_LS is the basis of our approach. In order to achieve the dynamic minimum cost benchmarking, we also need to calculate the PSS for general DDGs (or DDG segments). The PSS of a general DDG (or DDG segment) can be a high dimension space, because the DDG may have branches where there may be more than one \(X\) and/or \(V\) values that determine the MCSS of the DDG. Although a general DDG’s PSS is different from the DDG_LS’s PSS, they have similar properties and can be calculated with similar algorithms. In this sub-section for the
ease of understanding, we first investigate the PSS of a DDG segment that only has two branches, and then extend it to a general DDG.

5.2.3.1 Three dimension PSS of DDG segment with two branches

Figure 5.18 illustrates an example of a DDG segment that has two branches. As we can see that because of two branches, the MCSS of the DDG segment is determined by three variables, which are $X_1$, $V_2$, $V_3$. Hence the solution space of this DDG segment is a three dimension space where every MCSS occupies some space. Similar to the solution space of DDG_LS, we can find the border of two MCSSs, which is a partition plane in the three dimension solution space. For example, we assume that $S_{h,i,j}$ and $S_{h',i',j'}$ be two adjacent MCSSs in the solution space, where $SCR_{h,i,j} < SCR_{h',i',j'}$. The first and last stored datasets of these two strategies are in the positions as shown in Figure 5.18. The equation of the partition plane is

$$\left( \frac{1}{h-1} \sum_{k=h'}^{h-1} v_k \right) * X_1 + \left( \sum_{k=i+1}^{j} x_k \right) * V_2 + \left( \sum_{k=j+1}^{j} x_k \right) * V_3 = SCR_{i',j'} - SCR_{i,j}$$

To simplify the presentation of the equation, we introduce two new notations:

1) $\sum_{i}^{j} v_i = \begin{cases} \sum_{k=i}^{j-1} v_k, & d_i \rightarrow d_j \\ \sum_{k=j}^{j-1} v_k, & d_j \rightarrow d_i \end{cases}$ and 2) $\sum_{i}^{j} x_i = \begin{cases} \sum_{k=i+1}^{j} x_k, & d_i \rightarrow d_j \\ \sum_{k=j+1}^{j} x_k, & d_j \rightarrow d_i \end{cases}$

![Figure 5.18 DDG segment with two branches](image-url)
Similar to the DDG_LS, the equation of the partition plane also has different forms according to the positions of the start and end datasets of the two MCSSs. In general, for the DDG segment with two branches, given two MCSSs: 1) $S_p$ with the first stored dataset $d_{p1}$ and last stored datasets $d_{p2}, d_{p3}$; 2) $S_q$ with the first stored dataset $d_{q1}$ and last stored datasets $d_{q2}, d_{q3}$; and $SCR_p < SCR_q$. We have the standard form of the partition plane as following:

$$Bx_1 \left( \sum_{p_1} x \right) * X_1 + Bv_2 \left( \sum_{p_2} x \right) * V_2 + Bv_3 \left( \sum_{p_3} x \right) * V_3 = SCR_q - SCR_p$$

$$Bx_1 = \begin{cases} -1, & d_{p1} \rightarrow d_{q1} \\ 0, & d_{p2} = d_{q2} \\ 1, & d_{q1} \rightarrow d_{p1} \end{cases} \quad Bv_2 = \begin{cases} -1, & d_{q2} \rightarrow d_{p2} \\ 0, & d_{p2} = d_{q2} \\ 1, & d_{p2} \rightarrow d_{q2} \end{cases} \quad Bv_3 = \begin{cases} -1, & d_{q3} \rightarrow d_{p3} \\ 0, & d_{p3} = d_{q3} \\ 1, & d_{p3} \rightarrow d_{q3} \end{cases}$$

Similar to the DDG_LS, the DDG segment with two branches also has a PSS, in which the partition planes of the MCSSs intersect with each other and partition the solution space into different spaces. For any given values of $X_1, V_2, V_3$, we can locate an MCSS in the PSS for storing the DDG segment, if we know the distribution of MCSSs in the PSS. The three dimension PSS has similar properties as the PSS of DDG_LS. In order to calculate the PSS, we introduce another two lemmas, which describe important properties of the intersection lines and points in the three dimension PSS.

**Lemma 5.2:** In a three dimension PSS, for three MCSSs, if any two of them are adjacent with each other, then the three partition planes intersect in one line.

Figure 5.19 illustrates an example of Lemma 5.2. In Figure 5.19 (a), $S_a, S_b, S_c$ are three MCSSs of a DDG segment with two branches. We assume that $SCR_a < SCR_b < SCR_c$ and the start and end datasets of the three MCSSs have the following relationships: $d_{c1} \rightarrow d_{q1} \rightarrow d_{q1}, \ d_{a2} \rightarrow d_{b2} \rightarrow d_{c2}, \ d_{a3} \rightarrow d_{b3} \rightarrow d_{c3}$. Then we have three partition planes of $S_a, S_b, S_c$ as follows:
\[ P < S_a, S_b > : \left( \sum_{a_1}^{b_1} \right) \times X_1 + \left( \sum_{a_2}^{b_2} \right) \times V_2 + \left( \sum_{a_3}^{b_3} \right) \times V_3 = SCR_b - SCR_a \]

\[ P < S_b, S_c > : \left( \sum_{b_1}^{c_1} \right) \times X_1 + \left( \sum_{b_2}^{c_2} \right) \times V_2 + \left( \sum_{b_3}^{c_3} \right) \times V_3 = SCR_c - SCR_b \]

\[ P < S_a, S_c > : \left( \sum_{a_1}^{c_1} \right) \times X_1 + \left( \sum_{a_2}^{c_2} \right) \times V_2 + \left( \sum_{a_3}^{c_3} \right) \times V_3 = SCR_c - SCR_a \]

As shown Figure 5.19 (b), \( P < S_a, S_b > \) is the partition plane of \( S_a \) and \( S_b \); \( P < S_b, S_c > \) is the partition plane of \( S_b \) and \( S_c \); \( P < S_a, S_c > \) is the partition plane of \( S_a \) and \( S_c \). According to Lemma 5.2, the three partition planes intersect in one line \( L < S_a, S_b, S_c > \).

**Figure 5.19 Example of Lemma 5.2**

**Lemma 5.3:** In a three dimension PSS, for four MCSSs, if any three of them intersect in a different line, then the four intersection lines intersect at one point.
Figure 5.20 illustrates an example of Lemma 5.9. In Figure 5.20, $S_a$, $S_b$, $S_c$, $S_e$ are four MCSSs in the PSS and the partition planes denote the borders of the occupied spaces by the MCSSs. We assume that $L_{AB}$ (the line passing point $A$ and point $B$ in Figure 5.20) be the intersection line of $S_a$, $S_b$, $S_c$, $L_{AC}$ be the intersection line of $S_a$, $S_c$, $S_e$, $L_{AD}$ be the intersection line of $S_a$, $S_b$, $S_e$, $L_{AE}$ be the intersection line of $S_b$, $S_c$, $S_e$. According to Lemma 5.9, the four intersection lines intersect at point $A$.

![Diagram](image)

**Figure 5.20 Example of Lemma 5.3 - Four MCSSs’ intersection in a three dimension PSS, viewed from different angles**

### 5.2.3.2 High dimension PSS of a general DDG

After the two and three dimension illustration and description for helping understanding, we now discuss the general case. In a general DDG segment, there may exist multiple branches, hence there are more variables (i.e. more $X$ and $V$ dimensions) that impact the MCSS of the DDG segment. This makes the general DDG segment’s PSS a high dimension space, where the number of the dimensions is the total number of different $X$ and $V$ variables. In an $n$ dimension PSS, every MCSS occupies some $n$ dimension space, where we can calculate the border of every two MCSSs in the similar way as the three dimension PSS.

For an $n$ dimension PSS, we assume that there be $m$ branches with preceding datasets (i.e. different $X$ dimensions), hence $n-m$ branches with succeeding datasets (i.e. different $V$ dimensions). Given two MCSSs: 1) $S_p$ with the first stored datasets
\(d_{p,1}, d_{p,2}, \ldots, d_{p,m}\) in the \(m\) different \(X\) dimension branches and the last stored datasets \(d_{p,(m+1)}, d_{p,(m+2)}, \ldots, d_{p,n}\) in the \(n-m\) different \(V\) dimension branches; 2) \(S_q\) with the first stored datasets \(d_{q,1}, d_{q,2}, \ldots, d_{q,m}\) in the \(m\) different \(X\) dimension branches and the last stored datasets \(d_{q,(m+1)}, d_{q,(m+2)}, \ldots, d_{q,n}\) in the \(n-m\) different \(V\) dimension branches; and \(\text{SCR}_p < \text{SCR}_q\). Then, the border of \(S_p\) and \(S_q\) in the \(n\) dimension space is:

\[
\sum_{i=1}^{m} \left( Bx_i \ast \left( \sum_{p_i} \right) \ast X_i \right) + \sum_{j=m+1}^{n} \left( Bv_j \ast \left( \sum_{p_j} \right) \ast V_j \right) = \text{SCR}_q - \text{SCR}_p
\]

\[
Bx_i = \begin{cases} 
-1, & d_{p_i} \rightarrow d_{q_i} \\
0, & d_{p_i} = d_{q_i} \\
1, & d_{q_i} \rightarrow d_{p_i}
\end{cases} \quad Bv_j = \begin{cases} 
-1, & d_{q_j} \rightarrow d_{p_j} \\
0, & d_{p_j} = d_{q_j} \\
1, & d_{p_j} \rightarrow d_{q_j}
\end{cases}
\]

From the equation above, we can see that the border of two MCSSs in an \(n\) dimensions PSS is an \(n\)-variable linear equation, which is an \((n-1)\) dimension space itself. In order to calculate the PSS of a general DDG segment, we need to investigate the intersections of the MCSSs in the \(n\) dimension space. We generalise Lemmas 5.1-5.3 to the \(n\) dimension PSS of a general DDG segment and propose Theorem 5.6 as follows.

**Theorem 5.6:** In an \(n\) dimension PSS, for \(i\) MCSSs where \(i \in \{2,3,\ldots,(n+1)\}\), if any \((i-1)\) of the \(i\) MCSSs intersect in a different \((n-i+2)\) dimension space, then the \(i\) MCSSs intersect in an \((n-i+1)\) dimension space.

Based on Theorem 5.6, given the initial MCSS set of a general DDG segment (i.e. \(S_{\text{ini}}\)), we can design an algorithm to calculate the PSS in the similar way as the algorithm for calculating the PSS for DDG_LS. In Section 5.2.4, we will introduce how to derive \(S_{\text{ini}}\) of a general DDG segment without calling the CTT-SP algorithm on it. For a PSS with \(n_d\) dimensions, the border of MCSSs are \(n_d\)-variable linear equations and we need to solve the \(n_d\)-variable linear equations system to calculate an intersection point in the solution space which has a time complexity of \(O(n_d^3)\). Hence the time complexity of calculating a general DDG segment’s PSS is \(n_d^3\) times of the complexity for calculating the DDG_LS’s PSS, which is \(O(n_i^3 n_d^3)\). Similarly,
locating the MCSS in the high dimension PSS with given $X$ and $V$ values is also $n_3^d$ times complex than locating a MCSS in the two dimension PSS, which is $O(n^s n_3^d)$.

5.2.4 Dynamic on-the-fly Minimum Cost Benchmarking

The reason that we calculate the PSS for DDG segment is for dynamic minimum cost benchmarking. The philosophy of our approach is that we merge the PSSs of the DDG LSs to derive the PSS of the whole DDG and save all the calculated PSSs along this process. Taking advantage of the pre-calculated results (i.e. the saved PSSs), whenever the application cost changes, we only need to recalculate the local DDG LS’s PSS and quickly derive the new minimum cost benchmark for the whole DDG. By dynamically keeping the minimum cost benchmark updated, benchmarking requests can be instantly responded on the fly.

5.2.4.1 Minimum cost benchmarking by merging and saving PSSs in a hierarchy

To calculate the minimum cost benchmark with our approach, we need to merge the DDG segments’ PSSs in order to get the PSS of the whole DDG, from which we can locate the MCSS. To merge the PSSs of two DDG segments, we need to introduce another theorem.

Theorem 5.7: Given DDG segment $\{d_1, d_2, \ldots, d_m\}$ with PSS$_1$, DDG segment $\{d_{m+1}, d_{m+2}, \ldots, d_n\}$ with PSS$_2$, and the merged DDG segment $\{d_1, d_2, \ldots, d_m, d_{m+1}, d_{m+2}, \ldots, d_n\}$ with PSS. Then we have:

$$\forall S \in \text{PSS} \Rightarrow \left\{ S = S_1 \cup S_2, \quad S_1 \in \text{PSS}_1 \quad S_2 \in \text{PSS}_2 \right\}$$

$$\text{SCR} = \text{SCR}_1 + \left( \sum_{k=j+1}^{m} x_k \right) \ast \left( \sum_{k=m+1}^{i-1} v_k \right) + \text{SCR}_2$$

where $d_j$ is the last stored dataset in the first DDG segment and $d_i$ is the first stored dataset in the second DDG segment.

Theorem 5.7 tells us that 1) the MCSSs in a larger DDG segment’s PSS (i.e. $S$) are combined by the MCSSs in its sub-DDG segments’ PSSs (i.e. $S_1$, $S_2$), which
means that we can calculate the PSS of the larger DDG segment by merging the PSSs of its sub-DDG segments and do not need to call the CTT-SP algorithm on the larger DDG segment; 2) the cost rate of the MCSS in the larger DDG segment (i.e. \(SCR\)) is the sum of cost rates of its sub-DDG segments’ MCSSs (i.e. \(SCR_1\), \(SCR_2\)) and a parameter which is 

\[ \left( \sum_{k=j+1}^{m} v_k \right) \left( \sum_{k=m+1}^{i} v_k \right) \].

This parameter indicates the cost rate compensation for the datasets in the connecting branches of the two sub-DDG segments, i.e. generation cost rate of datasets in DDG segment \(\{d_{j+1}, d_{j+2}, \ldots d_m\}\) for regenerating datasets in DDG segment \(\{d_{m+1}, d_{m+2}, \ldots d_i\}\). Figure 5.21 further illustrates an example of Theorem 5.7 to merge two linear DDG segments.

![Figure 5.21 Example of merging two linear DDG segments](image)

Figure 5.22 shows the pseudo code of merging two PSSs. In this algorithm, we first find the MCSS candidate set for the merged PSS (i.e. \(S_{\text{All}}\)) by combining the MCSSs in the two sub-PSSs (lines 1-7). During this process we also calculate the SCR for every MCSS (line 5) and find the upper bound for \(SCR_{\text{max}}\) (lines 6-7). Next, we eliminate the invalid MCSSs from \(S_{\text{All}}\), which includes two sub-steps: 1) deleting the MCSSs with invalid SCR values (lines 8-10); 2) calling the elimination algorithm (see Figure 5.14) to derive \(S_{\text{ini}}\) (line 11). Then we call the PSS calculation algorithm (see Figure 5.16) to calculate the PSS of the merged DDG segment (line 12). From the pseudo code, we can clearly see that the time complexity of merging two PSSs is the same as the calculation of the PSS, which is \(O(n_s^3 n_d^3)\).

To calculate the PSS of a general DDG in the cloud, we can calculate all the PSSs of its sub-DDG LSs and gradually merge them to derive the PSS of the whole DDG. In order to achieve dynamic benchmarking, we need to save not only PSSs of
the DDG_Ls, but also PSSs calculated during the merging process. In our approach, we use a hierarchy data structure to save all the PSSs of a DDG, where an example of saving the PSS of a DDG with three sub-DDG_Ls is shown in Figure 5.23.

Algorithm: Merge PSSs
Input: PSS1 of DDG segment \{d1, d2, … dm\}
PSS2 of DDG segment \{dm+1, dm+2, … dm\}
Output: PSS for the merged DDG segment

1. S_All = \emptyset, SCR_max = \infty
2. for ( every MCSS S’ in PSS1 )
3.     for ( every MCSS S” in PSS2 )
4.         Add S = S’ \cup S” to S_All
5.         SCR = SCR’ + \left( \sum_{k=j+1}^{m} r_{k} \right) * \left( \sum_{k=j+1}^{m} r_{k} \right) + SCR”;
6.         if ( the first and last datasets in all the branches are all stored in S and SCR < SCR_max )
7.             SCR_max = SCR;
8.     for ( every MCSS S in S_All )
9.         if ( SCR > SCR_max )
10.             Delete S from S_All;
11.     S_init = Eliminate S_All (S_All); \quad \text{//O(n^3)}
12.     PSS = Calculate PSS (S_init); \quad \text{//O(n^3/\alpha^3)}
13.     Return PSS;

Figure 5.22 Pseudo code for merging PSSs

In the PSS hierarchy, the level indicates the number of DDG_Ls merged in the PSS of the DDG segments at that level. For example, in Figure 5.23, the DDG_Ls’ PSSs are saved at Level 1 of the hierarchy. Level 2 saves the PSSs of the DDG segments, which are connected by two DDG_Ls, e.g. PSS12 is the PSS of DDG segment combined by DDG_LS1 and DDG_LS2. Level 3 saves the PSS of the whole DDG, where we can see that the number of the hierarchy levels equals the number of DDG_Ls in the whole DDG. Furthermore, there are links between the levels in the hierarchy. A link between two PSSs at Levels i and i+1 in the hierarchy means the corresponding DDG segment of the PSS at Level i+1 contains the DDG segment of the PSS at Level i, e.g. in Figure 5.23, there is a link between PSS1 and PSS12, because the DDG segment combined by DDG_LS1 and DDG_LS2 contains DDG_LS1.
Figure 5.23 Saving all the PSSs of a DDG in a hierarchy

In the hierarchy, the highest level (e.g. Level 3 in Figure 5.23) saves the PSS of the whole DDG. From this PSS we can derive the MCSS and the corresponding SCR of the whole DDG, which is the minimum cost benchmark (i.e. SCR) that we can either proactively report or instantly respond to benchmarking requests. Next, we will introduce how to dynamically keep this benchmark updated.

5.2.4.2 Updating of the minimum cost benchmark on the fly

Cloud is a dynamic environment. As time goes on, new datasets are generated in the cloud and the existing datasets’ usage frequencies may also change. Hence the minimum cost benchmark of storing the datasets would also change accordingly. By taking the advantage of the PSS hierarchy, we can dynamically calculate the new minimum cost benchmark on the fly. There are two situations that we need to deal with:

1) New datasets are generated in the cloud.

The algorithm pseudo code of calculating the new minimum cost benchmark of this situation is shown in Figure 5.24. Assuming that the new datasets be in a DDG_LS (if not, we take its sub-DDG_LS), first we add it to the whole DDG and calculate its PSS, denoted as \( PSS_{new} \) (lines 1-3). Next, for every MCSS in \( PSS_{new} \), we locate the corresponding MCSS from the original DDG’s PSS (lines 5-7) and calculate the cost rate of the whole DDG, i.e. SCR (line 8). Then, we find the minimum SCR as the new minimum cost benchmark for the whole DDG and the corresponding storage strategy as the new MCSS (lines 9-11). In this whole process,
we only need to calculate the PSS of the new DDG_LS, which is usually small in size, and the PSS of the original DDG has already been pre-calculated and saved in the hierarchy. Hence we can quickly update the minimum cost benchmark. For example, in Figure 5.25 (a), for the new DDG_LS, we calculate PSS and connect it with the existing PSS in the hierarchy to derive the updated minimum cost benchmark and the MCSS of the whole DDG.

Algorithm: Generate new datasets
Input: DDG_LS //New datasets
PSS //PSS of the whole DDG
Output: S //MCSS of the whole DDG
SCR //Updated minimum cost benchmark

01. S_All = Find S_All (DDG_LS);
02. S_init = Eliminate S_All (S_All); //O(n^2)
03. PSS_new = Calculate PSS (S_init); //O(n^3n^3)
04. SCR = \infty; S = \emptyset;
05. for (every S_i in PSS_new)
06. V = \sum_{i=1}^{n} v_i;
07. S_temp = PSS.Locate (0,...,0,V);
08. SCR_min = SCR_temp + \left( \sum_{i=1}^{n} x_i \right) * V + SCR_{i,j};
09. if (SCR > SCR_min)
10. { S = S_temp \cup S_{i,j};
11. SCR = SCR_min; }
12. Return S, SCR;

Figure 5.24 Pseudo code for calculating new minimum cost benchmark when new datasets are generated

After calculating the new minimum cost benchmark, we have to update the PSS hierarchy for the new DDG_LS. For every newly added PSS at Level i of the hierarchy (starting from Level 1 to the highest level), we find its connected DDG_LS in the whole DDG and connect them to form a new segment. We calculate the PSS of the new segment and add it to Level i+1 of the hierarchy as well as the corresponding links between the two levels. An example of updating the PSS hierarchy in this situation is shown in Figure 5.25 (a), where the shadowed PSSs are the new ones that we add to the hierarchy after adding PSS.
2) Existing datasets’ usage frequencies are changed.

In this situation, we first find the DDG_LS that contains the datasets whose usage frequencies are changed. As shown in the pseudo code in Figure 5.26, we also need to calculate the DDG_LS’s PSS at the beginning (lines 1-3). Then, we find the PSSs of the rest parts of the whole DDG except the changed DDG_LS and save them in a set, i.e. $PSS_{Set}$ (line 4). Next, for every MCSS in the new PSS (line 6), we calculate the $X$ and $V$ values (line 7) to locate the corresponding MCSSs of the DDG segments that are connected to the changed DDG_LS from PSSs in $PSS_{Set}$ (lines 8-17). We also calculate the corresponding cost rate of the whole DDG, i.e. SCR (line 18). Then, we find the minimum SCR as the updated minimum cost benchmark for the whole DDG and the corresponding storage strategy as the new MCSS (lines 19-20). In this whole process, when calculating the PSS of the changed DDG_LS, we only update the weights of some edges in the existing CTT and do not need to create a new one. Furthermore, the PSSs of DDG segments in $PSS_{Set}$ have already been pre-calculated and saved in the hierarchy. Hence we can quickly update the minimum cost benchmark. For example, in Figure 5.25 (b), we re-calculate $PSS_2$ for changed $DDG_LS_2$. To derive the updated minimum cost benchmark for the whole DDG, we...
connect new $PSS_2$ with $PSS_{1,3}$ and $PSS_4$ in $PSS_{Set}$, which are the rest parts of the whole DDG except $DDG_{LS_2}$.

**Algorithm:** Change usage frequency

**Input:**
- $DDG_{LS_i}$ //With the changed dataset
- The PSS hierarchy

**Output:**
- $S$ //MCSS of the whole DDG
- $SCR$ //Updated minimum cost benchmark

01. $S_{All} = \text{Find } S_{All}(DDG_{LS_i});$
02. $S_{ini} = \text{Eliminate } S_{All}(S_{All});$  //O($n^2$)
03. $PSS_{new} = \text{Calculate } PSS(S_{ini});$  //O($n^2m^2$)
04. $PSS_{Set} = \text{PSSs of the DDG segments connecting to } DDG_{LS_i};$
05. $SCR = \infty; S = S' = \Phi;$
06. for (every $S_{ij}$ in $PSS_{new}$)
07.   \[
   V = \sum_{i=1}^{n} v_i, \quad X = \sum_{k=j+1}^{m} x_k \;
   \]
08.   for (every $PSS_h$ in $PSS_{Set}$)
09.   if ($PSS_h$ is preceding to $PSS_{new}$)
10.      \[
         S_{temp} = PSS_h.\text{Locate}(0...0,V); \\
         SCR_{min} = SCR_{temp} + \left( \sum_{k=j+1}^{m} x_k \right) * X;
      \]
11.   else if ($PSS_h$ is succeeding to $PSS_{new}$)
12.      \[
         S_{temp} = PSS_h.\text{Locate}(0...0,X); \\
         SCR_{min} = SCR_{temp} + \left( \sum_{k=1}^{j} v_k \right) * V;
      \]
13.      else
14.         \[
         S_{temp} = PSS_h.\text{Locate}(0...0,X,V); \\
         SCR_{min} = SCR_{temp} + \left( \sum_{k=1}^{j} v_k \right) * X + \left( \sum_{k=j+1}^{m} x_k \right) * V;
      \]
15.      \[
         S' = S' \cup S_{temp}; \\
         SCR_{min} = SCR + SCR_{min} + SCR_{temp};
      \]
16.   if ($SCR > SCR_{min}$)
17.       \[
         S = S' \cup S_{i,j}; \\
         SCR = SCR_{min};
      \]
18. else
19.      \[
         S = S' \cup S_{i,j}; \\
         SCR = SCR_{min};
      \]
20. Return $S, SCR$

![Figure 5.26 Pseudo code for calculating new minimum cost benchmark when datasets’ usage frequencies are changed](image)

After calculating the new minimum cost benchmark, we also need to update the PSS hierarchy for the changed PSS. For every changed PSS at Level $i$ of the hierarchy (starting from Level 1 to the highest level), we find the PSSs at Level $i+1$ that are linked with it and update all these PSSs. An example of updating the PSS hierarchy in this situation is shown in Figure 5.25 (b), where the shadowed PSSs are the new ones that we need to update in the hierarchy after changing $PSS_2$.

In terms of efficiency, our approach can instantly respond to users’ benchmarking requests by keeping the minimum cost benchmark updated on the fly.
Whenever new datasets are generated and/or existing datasets’ usage frequencies are changed, our algorithm can quickly calculate the updated minimum cost benchmark in $O(n_s^3n_d^3)$ (see Figure 5.24 and Figure 5.26), where all the parameters are for the local DDG_LS, which are usually very small. The total time complexity of our benchmarking approach includes updating the hierarchy, which is used to save the PSSs. We use $m$ to denote the number of DDG_LS in the whole DDG. In the situation of new datasets generation, we need to add one new PSS to every level of the PSS hierarchy (see Figure 5.25 (a)), where the number of the levels equals to the number of DDG_LSs in the whole DDG, hence the time complexity is $O(mn_s^3n_d^3)$. In the situation of existing datasets’ usage frequencies changing, we have to recalculate more than one PSS (i.e. in the magnitude of $m$) in every level of the hierarchy (see Figure 5.25 (b)), hence the time complexity is $O(m^2n_s^3n_d^3)$.

In Chapter 7, we will use experiment results to further demonstrate this dynamic on-the-fly benchmarking approach.

### 5.3 Summary

In this chapter, we propose two minimum cost benchmarking approaches for scientific applications in the cloud. Benchmarking is to calculate the minimum cost rate of storing the application datasets in the cloud, which achieves the best trade-off between computation and storage. This benchmark can be utilised to evaluate the cost-effectiveness of all datasets storage strategies. Our two novel benchmarking approaches are summarised as follows.

The static on-demand benchmarking approach is suitable for the situation that only occasional benchmarking is requested. In this situation the benchmarking is a one-time only computation provided as an on-demand service. In this approach, the novel CTT-SP algorithm is designed, which solves a seemingly NP-hard problem in a polynomial time complexity.

The dynamic on-the-fly benchmarking approach is suitable for the situation that more frequent benchmarking is requested at runtime. In this situation, the benchmarking service is delivered on the fly to instantly respond to the
benchmarking requests. In this approach, we thoroughly investigated the issue of computation and storage trade-off and proposed a novel concept of Partitioned Solution Space (PSS) to save the pre-calculated MCSSs. By utilising the pre-calculated results, whenever the application cost changes in the cloud, we can quickly calculate the new minimum cost benchmark. By dynamically keeping the benchmark updated, benchmarking requests can be instantly responded on the fly.
Chapter 6

Cost-Effective Datasets Storage Strategies

Due to the pay-as-you-go model, we design cost-effective datasets storage strategies for users based on the trade-off between computation and storage in the cloud. Different from benchmarking, in practice, the minimum cost storage strategy (MCSS) may not be the ultimate goal for the applications, because storage strategies should be efficient for users to facilitate at runtime in the cloud and may need to take users’ tolerance of data accessing delay into consideration. This chapter is organised as follows.

In Section 6.1, by investigating users’ requirements of data accessing delay and users’ preference of storing some particular datasets, we introduce two new attributes of the datasets in DDG accordingly [90]. With the new attributes and corresponding mechanisms, the storage strategies can 1) guarantee that all the application datasets’ regenerations fulfill users’ tolerance of data accessing delay, and 2) allow users to store some datasets with a higher cost according to their preferences.

In Section 6.2, we design an innovative cost rate based storage strategy. In this strategy, we directly compare generation cost rate and storage cost rate for every dataset to decide its storage status. The strategy can guarantee that the stored datasets in the system are all necessary, and can dynamically check whether the regenerated datasets need to be stored, and if so, adjust the storage strategy accordingly. This
strategy is highly efficient with fairly reasonable cost effectiveness. This section is mainly based on our work presented in [87, 92].

In Section 6.3, we design an innovative local-optimisation based storage strategy. In this strategy, we divide the DDG with large number of application datasets into small linear segments (DDG_LS). By partially utilising an enhanced linear CTT-SP algorithm, we can find the MCSS for the DDG_LS satisfying users’ requirements. Hence we achieve the localised optimisation in the storage strategy. This strategy is highly cost-effective with very reasonable runtime efficiency. This section is mainly based on our work presented in [90].

6.1 Data Accessing Delay and Users’ Preferences in Storage Strategies

With the excessive computation and storage resources in the cloud, users can flexibly choose whether to store a dataset or not. If a generated application dataset has been deleted for saving the storage cost, we have to regenerate it whenever it needs to be reused. Regeneration causes not only the computation resources, but also a computation delay for accessing the data, i.e. waiting for the dataset to get ready. Furthermore, in some applications, users may have their own preferences to store some datasets even at a higher cost due to some reasons such as the need for immediate data access.

In order to deal with these issues, we introduce another two attributes to the datasets in the DDG. For a dataset $d_i$, the new attributes are denoted as: $<T_i, \lambda_i>$, where

- $T_i$ is time duration that denotes users’ tolerance of dataset $d_i$’s accessing delay. Users have tolerance of delay when they want to access a dataset that needs regeneration. $T_i$ is the time constrains of the datasets’ regeneration. In the storage strategy, the regeneration time of every deleted dataset cannot exceed its $T_i$. Especially, if $T_i$ is smaller than the generation time of dataset $d_i$ itself (i.e. $T_i < x_i / \text{Price}_{cpu}$), where $\text{Price}_{cpu}$ is the price of computation resources used to
regenerate $d_i$ in the cloud), then we have to store $d_i$, no matter how expensive $d_i$’s storage cost is.

- $\lambda_i$ represents users’ preference of storing dataset $d_i$, which is a value between 0 and 1. In the storage strategy, we multiply dataset $d_i$’s storage cost rate (i.e. $y_i$) by its $\lambda_i$, and use this modified value to compare with $d_i$’s generation cost rate (i.e. $\text{genCost}(d_i) \times v_i$) for deciding its storage status. The two extreme situations: $\lambda_i=0$ indicates no matter how large $d_i$’s storage cost is, it has to be stored; $\lambda_i=1$ indicates the storage status of $d_i$ only depends on its generation cost and storage cost in order to reduce the total system cost.

These two attributes are generic for the datasets storage strategies. How to set their values depends on the requirements of specific applications. For example, some applications may have fixed time constraints, such as the weather forecast application [59]. In this situation, the value for $T_i$ is set according to the starting time and finishing time (i.e. deadline) of the application. Furthermore, in some applications, users may want immediate access to a particular dataset. In this situation, the value for $\lambda_i$ of this dataset needs to be set as zero. With these two attributes, we design two new runtime storage strategies for different situations in the cloud.

### 6.2 Cost Rate Based Storage Strategy

In this storage strategy, for every dataset in the cloud, we directly compare the generation cost rate and storage cost rate of the dataset itself to decide its storage status. This strategy is highly efficient. The details of algorithms and cost-effectiveness analysis are described next in this section.

#### 6.2.1 Algorithms for the Strategy

We design three algorithms to handle all three situations in the cloud to decide the proper storage status of the application datasets. We analyse the time complexity of the algorithms in this sub-section and further evaluate the efficiency of this cost rate based strategy by experiments described in Section 7.3.2.
### 6.2.1.1 Algorithm for deciding newly generated datasets’ storage status

We assume $d_i$ be a newly generated dataset. The pseudo-code of this algorithm is shown in Figure 6.1.

First we add its information to the DDG (line 1). We add edges pointing to $d_i$ from its provenance datasets and initialise its attributes. As $d_i$ is new which obviously does not have a usage history yet, we use the average value in the system as the initial value for $d_i$’s usage frequency.

Next, we check whether $d_i$ should be stored or not (lines 2-10). First, we check if the generation time of $d_i$ can satisfy users’ tolerance of data accessing delay (line 2). If not, we store $d_i$ (line 3). Then we only compare the generation cost rate of $d_i$ with its storage cost rate multiplied by $\lambda_i$, which are $\text{genCost}(d_i) * v_i$ and $y_i * \lambda_i$ (line 5). If the generation cost rate is larger than the storage cost rate, we store $d_i$ (line 6), otherwise we delete $d_i$ (line 8).

From pseudo-code in Figure 6.1, we can see that the worst case time complexity of the algorithm is $O(n_a)$ (i.e. calculating $\text{genCost}(d_i)$ in line 2), where $n_a$ is the largest number of a dataset’s deleted predecessors in the DDG.

```plaintext
Algorithm: Decide storage status of a newly generated dataset
Input: Newly generated dataset $d_i$;
DDG ;
Output: $f_i$ ; //Storage status of $d_i$

01. add $d_i$’s information to DDG ;
02. if ( $\frac{\text{genCost}(d_i)}{\text{Price}_{cpu}} > T_i$ )
03. $f_i = "$stored"$ ; //decide to store $d_i$
04. else
05. { if ( $\text{genCost}(d_i) * v_i > y_i * \lambda_i$ )
06. $f_i = "$stored"$ ; //decide to store $d_i$
07. else
08. $f_i = "$deleted"$ ; //decide to delete $d_i$
09.}
10. Return $f_i$ ; //storage status of $d_i$
```

Figure 6.1 Algorithm for deciding newly generated datasets’ storage status
6.2.1.2 Algorithm for deciding stored datasets’ storage status due to usage frequencies change

We assume \( d_i \) be a stored dataset whose usage frequency is changed in the cloud. We need to recalculate its storage status. The pseudo-code of this algorithm is shown in Figure 6.2.

First, we check whether the generation time of \( d_i \) can satisfy users’ tolerance of data accessing delay (line 1). If not, we keep it stored (line 2). Because \( d_i \) is stored originally, the deletion will increase the generation cost and time of its deleted successors. Hence we need to further check whether the generation time of \( d_i \)’s deleted successors can satisfy users’ tolerance of data accessing delay (lines 4-7). If not, we keep \( d_i \) stored (line 6). Then we compare \( d_i \)’s generation cost rate with storage cost rate in order to decide its storage status (lines 8-11).

From pseudo-code in Figure 6.2, we can see that the worst case time complexity of the algorithm is \( O(n_a n_b) \) (lines 4 and 5), where \( n_a \) is discussed in Section 6.2.1.1 and \( n_b \) here is the largest number of a dataset’s deleted successors in the DDG.

```
Algorithm: Decide storage status of a stored dataset
Input: Stored dataset \( d_i \);
 DDG ;
Output: \( f_i \); //Storage status of \( d_i \)

01. if ( \( \text{genCost}(d_i)/\text{Price}_{opu} > T_i \) )
02.   Return \( f_i = \text{“stored”} \);
03. else
04.   for (every deleted successor \( d_k \) of \( d_i \))
05.     if ( \( \text{(genCost}(d_i) + \text{genCost}(d_k))/\text{Price}_{opu} > T_i \) )
06.       \( v += v_k; \)
07.     else
08.       if ( \( \text{genCost}(d_i) * v > y_i * \lambda_i \) )
09.         Return \( f_i = \text{“stored”} \);
10.     else
11.         Return \( f_i = \text{“deleted”} \);
```

**Figure 6.2 Algorithm for deciding stored datasets’ storage status**
6.2.1.3 Algorithm for deciding regenerated datasets’ storage status

We assume that $d_i$ be a deleted dataset in the cloud. When we regenerate it for reuse, we have to recalculate its storage status after the reuse. The pseudo-code of this algorithm is shown in Figure 6.3.

Because $d_i$ is originally a deleted dataset, the storage of $d_i$ reduces the generation cost of its deleted successors. We need to take this cost reduction into consideration when calculating $d_i$’s generation cost rate (lines 1-2). Next, we compare $d_i$’s generation cost rate with storage cost rate in order to decide its storage status (lines 3-8). Especially, if $d_i$ is stored, it will not need its stored predecessors (i.e. $\text{provSet}_i$) for regeneration and its stored successors’ generation costs are also reduced, hence these stored predecessors and successors may not need to be stored anymore. We need to recalculate their storage status (lines 5-6).

From pseudo-code in Figure 6.3, we can see that the worst case time complexity of the algorithm is $O(n_a n_b n_c)$ (lines 5 and 6), where $n_a$ and $n_b$ are discussed in Sections 6.2.1.1 and 6.2.1.2 respectively and $n_c$ here is the largest number of a dataset’s stored predecessors and successors in the DDG. The efficiency of running the strategy will be evaluated in Section 7.3.2.

Algorithm: Decide storage status of a regenerated dataset

Input: Regenerated dataset $d_i$ ;
DDG ;

Output: $f_i$ ;  //Storage status of $d_i$

01. for (every deleted successor $d_k$ of $d_i$)
02. $v += v_k$
03. if ( $\text{genCost}(d_i) * (v_i + v) > y_i * \lambda_i$ )
04. $f_i$ = “stored” ;
05. for (every stored predecessor and successor $d_j$ of $d_i$)
06. recalculate the storage status of $d_j$ ;
07. else
08. $f_i$ = “deleted” ;
09. Return $f_i$ ;

Figure 6.3 Algorithm for deciding regenerated datasets’ storage status
6.2.2  Cost-Effectiveness Analysis

To analyse the cost-effectiveness of this cost rate based storage strategy, we need to introduce the following lemma and theorem.

**Lemma 6.1:** The deletion of a stored dataset in the DDG does not affect the storage status of other stored datasets.

**Theorem 6.1:** If a deleted dataset is stored, only its adjacent stored predecessors and successors in the DDG may need to be deleted to reduce the application cost.

Lemma 6.1 and Theorem 6.1 guarantee that the datasets stored by the algorithms in our cost rate based storage strategy are all necessary, which means that the deletion of any dataset will bring cost increase of the application in the cloud. The cost-effectiveness of this strategy will be further evaluated by experiments in Section 7.3.1.

6.3  Local-Optimisation Based Storage Strategy

In this storage strategy, we utilise the linear CTT-SP algorithm presented in Section 5.1.1 and enhance it by incorporating the two new attributes $< T_i, \lambda_i >$ addressed in Section 6.1, so that it can find the MCSS for linear DDG segments with satisfying users’ tolerance of computation delay and preference on storage. We use the enhanced CTT-SP algorithm on the linear segments in the large DDG, which achieves a localised optimisation. The details of algorithms and cost-effectiveness analysis are described next in this section.

6.3.1  Algorithms and Rules for the Strategy

First, we introduce the enhanced CTT-SP algorithm. Then, we describe the local-optimisation based storage strategy with the rules of using the enhanced CTT-SP algorithms in different situations.
6.3.1.1 Enhanced CTT-SP algorithm for linear DDG

The linear CTT-SP algorithm is described in Section 5.1.1. In the algorithm, we have:

\[(\forall d_i, d_j \in DDG \land d_i \rightarrow d_j) \Rightarrow \exists e < d_i, d_j>,\]

and the weight of the edge, i.e. \(\omega < d_i, d_j>\), means “the sum of cost rates of \(d_i\) and the datasets between \(d_i\) and \(d_j\), supposing that only \(d_i\) and \(d_j\) be stored and rest of the datasets between \(d_i\) and \(d_j\) all be deleted”.

To incorporate the delay tolerance attribute \(T\), in the enhanced linear CTT-SP algorithm, the edge \(e < d_i, d_j>\) has to further satisfy the condition:

\[e < d_i, d_j> \Rightarrow \forall d_k \in DDG \land (d_i \rightarrow d_k \rightarrow d_j) \land \left\{ \frac{genCost(d_k)}{Price_{cpu}} < T_k \right\}.\]

With this condition, long cost edges may be eliminated from the CTT. It guarantees that in all storage strategies of the DDG found by the algorithm, for every deleted dataset \(d_i\), its regeneration time is smaller than \(T_i\).

To incorporate the users’ preference attribute \(\lambda\), in the enhanced linear CTT-SP algorithm, we set the weight of a cost edge in CTT as

\[\omega < d_i, d_j> = y_j * \lambda_j + \sum_{d_k \in DDG: d_i \rightarrow d_k \rightarrow d_j} \left( genCost(d_k) * v_k \right) \]

In Figure 6.4, we demonstrate a simple example of constructing the CTT for a DDG that only has three datasets by the enhanced linear CTT-SP algorithm supposing that all the edges satisfy the computation delay tolerance.
Based on the discussion above, we give the pseudo code of the enhanced linear CTT-SP algorithm in Figure 6.5. From the pseudo code, we can see that for a linear DDG with n datasets, we have to add a magnitude of $n^2$ edges to construct the CTT (lines 1-2). In this enhanced linear CTT-SP algorithm, before actually creating an edge (line 9), we check whether this edge can satisfy the condition of users’ tolerance of regeneration time delay (lines 3-8). Next, we calculate the weight of the edges (lines 10-17), where we add the users’ preference attribute $\lambda$ (line 16). For the longest edge, the complexity of calculating its weight is $O(n^2)$ (lines 11-15), so a total of $O(n^4)$. Next, the Dijkstra shortest path algorithm has the time complexity of $O(n^2)$ (line 18). Hence, the enhanced linear CTT-SP algorithm also has a worst case time complexity of $O(n^4)$, and by adding the two new attributes, the algorithm can find the MCSS of linear DDG that satisfies users’ tolerance of computation delay and preference on storage.
Algorithm: Enhanced Linear CTT-SP
Input: start dataset $d_{s}$; end dataset $d_{e}$; a linear DDG; //Including $d_{s}$ and $d_{e}$
Output: $S$; //MCSS of the DDG $SCR$; //Minimum cost benchmark

01. for (every dataset $d_{i}$ in DDG) //Create CTT
02.   for (every dataset $d_{i}$, where $d_{i} \rightarrow d_{j}$)
03.     genCost = 0;
04.     for (every dataset $d_{o}$, where $d_{o} \rightarrow d_{i} \rightarrow d_{j}$)
05.         genCost = genCost + $x_{o}$;
06.     if (genCost/$Price_{cpu} > T_{j,i}$)
07.         break for;
08.   else
09.     Create $e < d_{i}, d_{j}>$; //Create an edge
10.     weight = 0;
11.     for (every dataset $d_{m}$, where $d_{m} \rightarrow d_{i} \rightarrow d_{j}$)
12.         genCost = 0;
13.         for (every dataset $d_{m}$, where $d_{m} \rightarrow d_{i} \rightarrow d_{j}$)
14.             genCost = genCost + $x_{m}$;
15.             weight = weight + ($x_{m} + genCost$) * $v_{i}$;
16.             weight = weight + $y_{i}$ * $\lambda_{j}$;
17.     Set $\omega < d_{i}, d_{j} >$ = weight; //Set weight to an edge
18. $P_{min}<d_{s}, d_{e}>$ = Dijkstra Algorithm ($d_{s}, d_{e}, CTT$);
19. $S$ = set of datasets that $P_{min}<d_{s}, d_{e}>$ traversed;
20. $SCR = \left(\sum_{d_{s}\rightarrow d_{e}<d_{i}, d_{j}>} CostR_{j}\right)_{S}$;
21. Return $S, SCR$;

Figure 6.5 Pseudo-code of enhanced CTT-SP algorithm

6.3.1.2 Rules in the Strategy

Based on the enhanced linear CTT-SP algorithm, we introduce our local-optimisation based datasets storage strategy. The philosophy is to derive localised minimum costs instead of a global one with low time complexity for the strategy. The strategy contains the following four rules:

1. Given a general DDG, the datasets to be stored first are 1) the ones that users have no tolerance of computation delay on them and 2) the ones that users choose to store.

2. Then, the DDG is divided into separate sub DDGs by the stored datasets. For every sub DDG, if it is a linear one, we use the enhanced CTT-SP algorithm to find its storage strategy; otherwise, we find the datasets that have multiple direct predecessors or successors, and use these datasets as the partitioning points to divide it into sub linear DDG segments, as shown in Figure 6.6. Then
we use the enhanced linear CTT-SP algorithm to find their storage strategies. This is the essence of local optimisation.

3. When new datasets are generated in the system, they will be treated as a new sub DDG and added to the old DDG. Correspondingly, its storage status will be calculated in the same way as the old DDG.

4. When a dataset’s usage rate is changed (by either system administrator or users), we will re-calculate the storage status of the sub linear DDG that contains this dataset.

![Figure 6.6 Dividing a DDG into sub linear DDGs](image)

In the strategy introduced above, the computation time complexity is well controlled within $O(m*n_i^4)$ by dividing the general DDG into sub linear DDG segments, where $m$ is the number of the sub linear DDGs and $n_i$ is the number of datasets in the sub linear DDG segments. Hence our strategy has a very reasonable computation complexity at runtime of the system which depends on the size of the sub linear DDGs. The efficiency of running the strategy will be evaluated in Section 7.3.2. Meanwhile, by utilising the CTT-SP algorithm, we guarantee that every sub linear DDG segment in the general DDG is stored with its MCSS, hence achieves the local optimisation.

### 6.3.2 Cost-Effectiveness Analysis

To analyse the cost-effectiveness of the local-optimisation based storage strategy, we need the following theorem.
**Theorem 6.2:** Given a DDG and assume $S$ be the MCSS of the DDG. If $d_p \in S$ and $d_p$ divides the DDG into:

\[
\begin{align*}
DDG_1 & = \{ d_j \mid d_j \in DDG \land d_j \rightarrow d_p \} \\
DDG_2 & = \{ d_k \mid d_k \in DDG \land d_p \rightarrow d_k \}
\end{align*}
\]

then $S_1$ and $S_2$ are the MCSSs of $DDG_1$ and $DDG_2$ respectively, where $S_1 = S \cap DDG_1$ and $S_2 = S \cap DDG_2$.

Based on Theorem 6.2, we analyse the difference between the cost rate of merging two linear DDG segments together by our strategy and the minimum cost rate (i.e. the benchmark addressed in Chapter 5).

Assume that linear DDG$_1$ with datasets $\{d_1, d_2, \ldots, d_u\}$ be stored with the minimum cost strategy $S$, which is calculated by the CTT-SP algorithm, and linear DDG$_2$ with datasets $\{d'_1, d'_2, \ldots, d'_v\}$ is added after DDG$_1$, as shown in Figure 6.7. We assume that $S$ be the MCSS of the merged DDG.

![Figure 6.7 Two merging DDG_LSs](image)

According to the local-optimisation based strategy, we calculate the storage strategy $S_2$ of DDG$_2$ separately, also by the CTT-SP algorithm. There are two situations as follows:

1) If the last dataset $d_u$ in DDG$_1$ is a stored dataset, the cost rate of the two merged DDGs in our strategy is the minimum cost rate, where

\[
\left( \sum_{d_i \in DDG_1} CostR_i \right)_{S_1} + \left( \sum_{d_i \in DDG_2} CostR_i \right)_{S_2} = \left( \sum_{d_i \in DDG_1 \cup DDG_2} CostR_i \right)_{S_2}.
\]

This can be proved by directly utilisation of the definition of SCR (Formula (4.4) in Section 4.3). Hence, local-optimisation based strategy is the MCSS of the DDG in this situation.
2) If the last dataset $d_k$ in DDG$_1$ is a deleted dataset, as shown in Figure 6.7, the CTT-SP algorithm on DDG$_2$ will start from $d_k$, which is the last stored dataset in DDG$_1$. Hence we have the MCSS $S'_2$ of the set of datasets after $d_k$, which is

$$DDG'_2 = \{ d_i | d_i \in (DDG_1 \cup DDG_2) \land d_k \rightarrow d_i \}.$$  

Because DDG$_1$ is stored with the minimum cost strategy and $d_k$ is a stored dataset, from Theorem 6.2, we can get $S'_1$ which is the MCSS of the set of datasets before $d_k$, which is

$$DDG'_1 = \{ d_i | d_i \in DDG_1 \land d_i \rightarrow d_k \}.$$  

Hence, the difference of the cost rate between our strategy and the minimum cost strategy of the merged DDG is:

$$\left( \sum_{d_i \in DDG'_1} CostR_i \right)_{S'_1} + y_k + \left( \sum_{d_i \in DDG'_2} CostR_i \right)_{S'_2} - \left( \sum_{d_i \in DDG_1 \cup DDG_2} CostR_i \right)_S < y_k - \left( genCost(d_k) \right) \ast v_k$$

This is because $S'_1$ and $S'_2$ are the minimum cost strategies of $DDG'_1$ and $DDG'_2$. Furthermore because $d_k$ is a deleted dataset according to the minimum cost strategy $S$ of the merged DDG (if $d_k$ is a stored dataset, then $S'_1 \cup S'_2 = S$ according to Theorem 6.2), we can come to a conclusion that the difference of the cost rate between our strategy and the minimum cost strategy is less than

$$y_k - genCost(d_k) \ast v_k$$

For more complex scenarios of merging DDGs in our strategy, as indicated in Figure 6.8, we have similar conclusions. In Section 7.3.1, we will use experiment results to further demonstrate the cost-effectiveness of the local-optimisation based storage strategy.
6.4 Summary

In this chapter, we present two novel datasets storage strategies that can be facilitated at runtime in the cloud. Besides taking into consideration of users’ tolerance of computation delay and preference of storing some datasets at higher cost, the two strategies provide different levels of efficiency and cost-effectiveness to meet the requirements of different applications. Specifically, the cost rate based strategy is highly efficient with fairly reasonable cost-effectiveness, and the local-optimisation based strategy is highly cost-effective with very reasonable efficiency.
Chapter 7
Experiments and Evaluations

In this chapter, we evaluate the proposed benchmarking approaches and storage strategies by experiments on our SwinCloud environment [60]. In Section 7.1, we introduce SwinCloud, which is a private cloud in Swinburne University of Technology. In Sections 7.2 and 7.3, we conduct general random experiments to evaluate the overall performance of our benchmarking approaches and storage strategies presented in Chapters 5 and 6 respectively. In Section 7.4, we describe a specific case study of the real world pulsar searching application which is the motivating example described in Section 3.1, in which our benchmarking approaches and storage strategies are illustrated.

7.1 Experiment Environment

SwinCloud is a cloud computing simulation environment. The architecture of SwinCloud is depicted in Figure 7.1. It is built on the computing facilities in Swinburne University of Technology and takes advantage of the existing SwinGrid systems [85]. For example, the Swinburne Astrophysics Supercomputer Node (http://astronomy.swin.edu.au/supercomputing/) comprises 145 Dell Power Edge 1950, each with: 2 quad-core Clovertown processors at 2.33 GHz (each processor is 64-bit low-volt Intel Xeon 5138), 16 GB RAM and 2 x 500 GB drives. We install VMWare [6] on SwinGrid, so that it can offer unified computing and storage resources. Utilising the unified resources, we set up data centres that can host
applications. In the data centres, Hadoop [3] is installed that can facilitate the Map-Reduce [32] computing paradigm and distributed data management.

![SwinCloud Infrastructure](image)

**Figure 7.1 SwinCloud Infrastructure**

### 7.2 Evaluation of Minimum Cost Benchmarking Approaches

In this section, we evaluate the minimum cost benchmarking approaches proposed in Chapter 5 by conducting general random simulations on SwinCloud. In Section 7.2.1, we evaluate the cost-effectiveness of the minimum cost benchmark by comparing it with some intuitive storage strategies. In Section 7.2.2, we evaluate the efficiency of the two different benchmarking approaches.

#### 7.2.1 Cost-Effectiveness Evaluation of the Minimum Cost Benchmark

To evaluate the cost-effectiveness of the minimum cost benchmark, we compare it with some representative and intuitive storage strategies, which are:
1. Store none dataset: delete all the generated datasets in the cloud, and regenerate them whenever needed.

2. Store all datasets: store all the application datasets in the cloud.

3. Generation cost based strategy: store the datasets that incur the highest generation costs.

4. Usage based strategy: store the datasets that are most frequently used.

For evaluation, we generate random DDGs and derive the minimum cost benchmark via the benchmarking approaches. We run the above four strategies on the DDG and compare the application costs with the minimum cost benchmark. From the large number of test cases in our experiment, we choose and present one as the representative in this sub-section.

In this case, we use a DDG with 50 datasets, each with a random size ranging from 100GB to 1TB. The dataset generation time is also random, ranging from 1 hour to 10 hours. The usage frequency is again random, ranging from once per day to once per 10 days. The prices of cloud services follow Amazon clouds’ cost model, i.e. $0.1 per CPU instance-hour for computation and $0.15 per gigabyte per month for storage. We run our benchmarking algorithm on this DDG to calculate the MCSS and the minimum cost benchmark, where 9 of the 50 datasets are chosen to be stored. We evaluate this minimum cost benchmark by comparing the total application cost of the other storage strategies introduced above.

Figure 7.2 shows the comparison of the minimum cost benchmark with the generation cost based strategy. We compare the total application costs over 30 days of the strategies that store different percentages of datasets based on the generation cost, and the minimum cost benchmark. The two extreme strategies of storing all the datasets and deleting all the datasets are also included. In Figure 7.2, we can clearly see the cost effectiveness of different strategies comparing with the benchmark. In this case, storing top 10% generation cost datasets turns out to be the most cost-effective strategy, which is still much higher (about 170%) than the minimum cost benchmark.
Figure 7.2 Cost-effectiveness evaluation by comparing with the generation cost based strategy

Figure 7.3 Cost-effectiveness evaluation by comparing with the usage based strategy

Then we compare the minimum cost benchmark with the usage based strategy. We still run simulations of strategies that storing different percentages of datasets based on their usage frequencies. Figure 7.3 shows the comparison of the total application costs over 30 days, where we can clearly see the cost effectiveness of different strategies comparing with the benchmark. Also, the strategy of storing top 10% often used datasets turns out to be the most cost-effective one in this case. Comparing to Figure 7.2, although the usage based strategy is more cost-effective than generation cost based strategy, it is again still much higher (about 70%) than the minimum cost benchmark.
From the experiments above, we can see the cost-effectiveness of the minimum cost benchmark, which serves very well as the benchmark for evaluating any storage strategies.

### 7.2.2 Efficiency Evaluation of Two Benchmarking Approaches

In Chapter 5, we develop two different benchmarking approaches according to different users’ requirements, i.e. static on-demand approach and dynamic on-the-fly approach. In this sub-section, we evaluate the efficiency of these two approaches.

In the simulation, the same random parameters in Section 7.2.1 are used to generate the DDG_LS with 50 datasets. The prices of cloud services again follow Amazon clouds’ cost model. To evaluate the two approaches, we start from one DDG_LS and gradually add new DDG_LSs to it (i.e. from \(m=1\) to \(m=20\)). For the DDGs with different sizes, we calculate the updated benchmark of the whole DDG with both the static on-demand benchmarking approach and the dynamic on-the-fly benchmarking approach. Figure 7.4 shows the comparison of CPU time consumed by the two benchmarking approaches.

From Figure 7.4 we can see that the on-demand benchmarking approach is not efficient to keep the minimum cost benchmark updated at runtime. The computation time increases dramatically as the datasets number increases. This is because whenever the cost is changed in the cloud, either because of the new datasets generation or the changes of existing datasets’ usage frequencies, we need to call the CTT-SP algorithm (see Section 5.1.3) for the whole DDG to calculate the new minimum cost benchmark. In contrast, for the dynamic benchmarking approach, as we can see from the zoom-in chart (bottom plane) in Figure 7.4, the time for calculating new minimum cost benchmark is in the magnitude of seconds in general, hence much more efficient. This is because we take advantage of the pre-calculated PSSs that are saved in the hierarchy (see Section 5.2.4) and only need to recalculate the PSS of the local DDG_LS to derive the new benchmark. Hence, the complexity of calculating the new benchmark is more or less independent of the size of the DDG.
Figure 7.4 Efficiency comparison of two benchmarking approaches

More specifically, the zoom-in chart (bottom plane) in Figure 7.4 shows that the time for calculating new minimum cost benchmark in the situation of datasets’ usage frequencies changing is less than new datasets generation. This is because when new datasets are generated, we need to create a new CTT for them to calculate the new PSS, whereas when existing datasets’ usage frequencies change in a DDG_LS, we only need to update the weights of the changed edges in the existing CTT instead of creating a new one to recalculate the PSS.

From Figure 7.4, we also note that after the calculation of new benchmark, the update of the PSS hierarchy takes some computation time. More specifically, the computation time of updating the PSS hierarchy for new datasets generation increases in a linear manner as the number of DDG_LS grows, because we need to add a new PSS to every level of the PSS hierarchy, where the number of the levels equals the number of segments in the whole DDG as presented in Section 5.2.4.2. However, in the situation of datasets’ usage frequencies changing, the computation time increases faster. This is because the newly generated datasets only have
preceding datasets in the original DDG, while the corresponding DDG_LS of the datasets whose usage frequencies are changed has both preceding and succeeding datasets in the original DDG. According to the rules of updating the PSS hierarchy presented in Section 5.2.4.2 (see Figure 5.25), we have to recalculate more than one PSS in every level of the hierarchy in the datasets’ usage frequencies changing situation.

Next, we conduct more specific experiments to analyse the impact on the efficiency of the benchmarking approaches. For the static on-demand approach, the efficiency depends on the number of datasets in the DDG, which is already illustrated in Figure 7.4. Hence, we mainly investigate the dynamic on-the-fly approach.

PSS is the basis of the dynamic on-the-fly approach, where the efficiency of calculating PSSs plays a decisive role in the overall performance. As discussed in Section 5.2.3.2, the time complexity of calculating PSS is determined by the number of dimensions of the PSS and the number of MCSSs in the PSS. The number of dimensions of a PSS only depends on the structure of the DDG, whereas the number of MCSSs in a PSS may depend on more factors. Hence we mainly investigate the latter, i.e. which factors impact the MCSSs in PSS and how they impact the efficiency of calculating PSS. We also briefly analyse the impact of PSS’s dimensions on the efficiency at the end of this sub-section.

Figure 7.5 contains the number of MCSSs in the PSSs generated by the experiments demonstrated in Figure 7.4, where Figure 7.5 (a) shows that as the size of DDG increases, the number of MCSSs in its PSS does not increase in general, and Figure 7.5 (b) further shows 8 MCSSs in the PSS of a DDG_LS with 50 datasets in detail. From the figure we can see that the number of MCSSs in the PSS is 1) not correlated to, and 2) much smaller than the number of datasets in the DDG_LSs. This important fact guarantees the efficiency of the on-the-fly benchmarking approach, which is based on the algorithm of calculating PSSs.
Next, in the following experiments, we investigate the parameters of the DDG that impact the number of MCSSs in the PSS and their impacts on the efficiency of calculating the PSS. First, we investigate the datasets’ generation time. To demonstrate the impact, for every dataset in a DDG_LS, we multiply its generation time by a modification parameter (i.e. 0.5~2), which makes the generation time changing from half to double of its original value, with other parameters unchanged. With different modification parameters, we generate different modified DDG_LSs and calculate their PSSs. Figure 7.6 (a) demonstrates the number of MCSSs in the PSSs and corresponding CPU times of the calculation, where we can see that as the modification parameter increases (i.e. the generation time of datasets increase), the number of MCSSs in the PSS decreases. Furthermore, because of the fact that the smaller the datasets generation time is, the more datasets in the DDG_LS will be stored to reduce the total application cost. Therefore, we reach a conclusion that the more datasets in DDG_LS are stored, the fewer MCSSs are in the PSS. Figure 7.6 (a) also shows that as the number of MCSSs changes, the CPU time of calculating PSS does not change very much. Next, we investigate the datasets’ sizes and usage.
frequencies to see their impacts on the PSS. With same experiments for Figure 7.6 (a), we get similar results which are shown in Figure 7.6 (b) and (c). Based on these experiments we can see that for randomly generated DDG_LSs with different parameters, the PSSs can be efficiently calculated where the change of the parameters has limited impact on the efficiency of our approach.

![Figure 7.6 Impacts of DDG’s parameters on the performance of the dynamic on-the-fly benchmarking approach](image)

Another factor that may impact on the efficiency of calculating PSS is the number of dimensions of the PSS. As discussed in Section 5.2.3.2, the impact on time complexity of the PSS dimension number is in the same magnitude as the MCSSs number in the PSS (i.e. $O(n_d^3)$). Furthermore, in real applications the number of dimensions of the PSS (i.e. the branches in the DDG) is usually not very high. Hence
the efficiency impact is not significant on calculating the PSS. In Section 7.4, we will utilise our benchmarking approaches in a pulsar searching application that has a DDG_LS.

7.3 Evaluation of Cost-Effective Storage Strategies

In this section, we evaluate the two cost-effective storage strategies proposed in Chapter 6, i.e. the cost rate based strategy and the local-optimisation based strategy. In Section 7.3.1, we evaluate the cost-effectiveness of the two strategies by comparing to the minimum cost benchmark. In Section 7.3.2, we evaluate the efficiency of the two proposed storage strategies.

7.3.1 Cost-Effectiveness of Two Storage Strategies

To be consistent, we use the same DDG randomly generated with the parameters in Section 7.2.1 to conduct the experiment. We run the cost rate based strategy and local-optimisation based strategy on the DDG_LS with 50 datasets, and compare the application cost with the minimum cost benchmark and the intuitive storage strategies introduced in Section 7.2.1. To demonstrate the cost-effectiveness of the strategies comparing to the minimum cost benchmark, we do not consider users’ tolerance of computation delay and storage preference initially. Figure 7.7 illustrates the comparison of total application cost over 30 days.

From Figure 7.7 we can see that the cost rate based strategy is more cost effective than the generation cost based strategy and usage based strategy on storing the DDG_LS. The local-optimisation based strategy stores the datasets with the same cost with the minimum cost benchmark. This is because we treat the DDG_LS as the entire segment and directly utilise the enhanced CTT-SP algorithm on it. Next we do more simulations on larger general DDGs to further demonstrate the cost-effectiveness of the two proposed storage strategies.
Figure 7.7 Comparison of the total cost for different storage strategies

We still use the same random parameters to generate the DDG_LS with 50 datasets. In the same way as Section 7.2.2, we start from one DDG_LS and gradually add new DDG_LSs to it. Hence for the local-optimisation based strategy, every DDG_LS is a segment to utilise the enhanced CTT-SP algorithm. Different from the former simulations, we do not accumulate the total cost anymore; instead, we calculate the cost rate (average daily cost over 30 days) of storing all the datasets. This allows us to incorporate more simulation results in one figure for the better comparison purpose. The results are illustrated in Figure 7.8.

Figure 7.8 Comparison of the cost rate for different storage strategies

Figure 7.8 shows the increases of the cost rates of different strategies as the number of datasets grows in the DDG. The results are consistent with formal
experiments, where we can see the “store none” and “store all” strategies are very cost ineffective, since their cost rates grow fast as the datasets number grows. The cost rate based strategy is better than both the generation cost based strategy and usage based strategy. The local-optimisation based strategy is the most cost-effective datasets storage strategy, which is very close to the minimum cost benchmark. Hence the local-optimisation based strategy is highly cost effective.

As discussed in Section 6.1, cost is not the only issue for storing application datasets in the cloud, and users may have a certain degree of tolerance for data accessing delay and have preference of storing some datasets with a higher cost. The storage of these datasets may well incur extra application cost, hence has some impact on the cost-effectiveness of the storage strategies. The more datasets users choose to store, the less datasets the storage strategy can apply to, hence the datasets storage strategy would become less cost effective. Next, we ran another set of simulations on a 200 datasets DDG with different percentages of the datasets that are stored in the cloud based on users’ preferences rather than cost. The rest of the parameter setting is the same as previous simulations. The results are shown in Figure 7.9.

![Change of daily cost (200 datasets in the DDG)](image)

**Figure 7.9 Impact on cost-effectiveness of the storage strategies**

From Figure 7.9 we can see that besides the two extreme strategies, i.e. store none dataset and store all datasets, all of the rest four strategies gradually become more cost ineffective as the percentage of users stored datasets increases. However,
the cost rate based strategy and local-optimisation based strategy proposed in this thesis are still more cost-effective than others.

7.3.2 Efficiency Evaluation of Two Storage Strategies

The storage strategies are designed for runtime utilisation in the cloud, hence they need to be efficient. In this sub-section, we evaluate the efficiency of the two proposed strategies by comparing their execution time to the original CTT-SP algorithm used for benchmarking.

To be consistent, we still use the randomly generated DDG in Figure 7.8’s experiment for this simulation, i.e. the large general DDG combined by DDG_LSs with 50 datasets \((n=50)\). We run the cost rate based strategy and the local-optimisation based strategy on the DDGs and compare their CPU time with the CTT-SP algorithm used for benchmarking. For the local-optimisation based strategy, the computation time not only depends on the number of datasets in the DDG, but also the partition of the DDG. Hence, we incorporate another two sets of simulations with different partition methods of the DDG, i.e. 1) we use DDGs that only have 5 linear segments \((m=5)\) and let the number of datasets in each segment grow; 2) we control the partition of the DDGs that every segment has at most 10 datasets \((n=10)\) and let the number of segments grow. The simulation results are shown in Figure 7.10.

![Figure 7.10 Efficiency comparisons of different storage strategies](image-url)
In Figure 7.10, as the number of datasets increases, we can see that the original CTT-SP algorithm is not efficient, where it takes more than 200 seconds to find the MCSS for the DDG with 200 datasets. Hence it can only be used for on-demand benchmarking. The cost rate based strategy is highly efficient. The local-optimisation based strategy is not as efficient as the cost rate based strategy, especially when the number of datasets in the segment is very large.

### 7.4 Case Study of Pulsar Searching Application

As introduced in Section 3.1, pulsar searching is a typical scientific application in astrophysics. In this section, we demonstrate how the pulsar case utilises our benchmarking approaches and storage strategies on storing the generated application datasets.

In the pulsar case, during the workflow’s execution on analysing ONE PIECE of the observation data, six datasets are generated. The DDG of these datasets is shown in Figure 7.11, as well as the sizes and generation times of these datasets. From Swinburne astrophysics research group, we understand that the “de-dispersion files” are the most useful generated dataset. Based on these files, many accelerating and seeking methods can be used to search pulsar candidates. Based on the scenario, we set the “de-dispersion files” to be used once every 4 days, and the rest of the datasets to be used once every 10 days. Furthermore, we also assume that the prices of cloud services follow Amazon clouds’ cost model.

![Diagram of pulsar searching application](image)

*Figure 7.11 DDG of pulsar searching application*
7.4.1 Utilisation of Minimum Cost Benchmarking Approaches

The utilisation of the static on-demand benchmarking approach is straightforward. We directly create the CTT on the DDG and find the MCSS, which is storing $d_2$, $d_4$, $d_6$ and deleting $d_1$, $d_3$, $d_5$. The minimum cost benchmark is $0.51 per day.

Next we demonstrate the utilisation of the dynamic on-the-fly benchmarking approach. As described in Section 3.1, there are two phases in execution of the workflow to generate the DDG: Files Preparation and Seeking Candidates, where in each phase three datasets are generated as a DDG_LS. Figure 7.12 demonstrates the PSS calculation of the two DDG_LSs and the merging process for the PSS of the whole DDG segment.

![Diagram](image)

Figure 7.12 PSSs of a DDG segment in the pulsar application
When datasets $d_1, d_2, d_3$ are generated as $DDG\_LS_1$, we calculate $PSS_1$. Next, when datasets $d_4, d_5, d_6$ are generated as $DDG\_LS_2$, we first calculate its PSS, denoted as $PSS_2$, then we locate the corresponding MCSS from $PSS_1$ and form the MCSS of the whole DDG segment which stores datasets $d_2, d_4, d_6$. Next we calculate the cost rate of the MCSS, which is again $0.51 \text{ per day for storing these six datasets}$. This cost rate is the minimum cost benchmark. After we derive the new benchmark, we need to merge $PSS_1$ and $PSS_2$ to derive the PSS of the whole DDG segment, which is saved in the hierarchy for further use.

### 7.4.2 Utilisation of Cost-Effective Storage Strategies

The storage strategies are utilised at runtime in the cloud. As time goes on, researchers may reuse the datasets and conduct new re-analysis on them, where new datasets are generated. Base on the scenario, we set that new datasets are generated on the 10th day and 20th day, indicated as sub DDG1 and sub DDG2 in Figure 7.13.

We run the two proposed cost-effective storage strategies on the DDG and compare the total application cost with the same storage strategies previously presented and minimum cost benchmark. The simulation results are shown in Figure 7.14.

---

**Figure 7.13 DDG of the pulsar application with new datasets generation**
From Figure 7.14 we can see that 1) the cost of the “store none dataset” strategy is a fluctuated line because in this strategy all the costs are computation cost of regenerating datasets. For the days that have fewer requests of the data, the cost is low, otherwise, the cost is high; 2) the cost of the “store all datasets” strategy is a polyline, because all the datasets are stored in the system that is charged at a fixed rate, and the inflection points only occur when new datasets are generated; 3-4) the costs of the generation cost based strategy and the usage based strategy are in the middle band, which are lower than the “store none dataset” and “store all datasets” storage strategies. The cost lines are slightly fluctuated because the datasets are partially stored; 5-6) the cost rate based strategy has a good performance and the most cost-effective datasets storage strategy is the local-optimisation based strategy which achieves storing the datasets with the minimum cost in this pulsar searching application. Table 7.1 shows how the datasets are stored with different strategies in detail.

![Figure 7.14 Cost-effectiveness comparisons of different storage strategies for storing pulsar case DDG](image)

Since the pulsar DDG shown in Figure 7.13 is not complicated, we can do some intuitive analyses on how to store the generated datasets. For the dataset of Accelerated De-dispersion Files, although its generation cost is quite high, comparing to its huge size, it is not worth storing them in the cloud. However, in the generation cost based strategy, these files are stored. For the final XML Files, they are
not very often used, but comparing to their high generation cost and small size, they should be stored. However, in the usage based strategy, these files are not stored. For the dataset of *De-dispersion Files*, by comparing its own generation cost rate and storage cost rate, the cost rate based strategy did not store it at the beginning, but store it after it is used in the regeneration of other datasets. In this case, the local-optimisation based strategy is the most cost-effective datasets storage strategy for storing the datasets, which achieves the minimum cost storage strategy.

**Table 7.1 Storage status of datasets in the pulsar application with different storage strategies**

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Extracted beam</th>
<th>De-dispersion files</th>
<th>Accelerated de-dispersion files</th>
<th>Seek results</th>
<th>Pulsar candidates</th>
<th>XML files</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Store none dataset</td>
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<td>Deleted</td>
<td>Deleted</td>
<td>Deleted</td>
<td>Deleted</td>
<td>Deleted</td>
</tr>
<tr>
<td>2) Store all datasets</td>
<td>Stored</td>
<td>Stored</td>
<td>Stored</td>
<td>Stored</td>
<td>Stored</td>
<td>Stored</td>
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<tr>
<td>3) Generation cost</td>
<td>Deleted</td>
<td>Stored</td>
<td>Stored</td>
<td>Deleted</td>
<td>Deleted</td>
<td>Stored</td>
</tr>
<tr>
<td>based strategy</td>
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<tr>
<td>4) Usage based strategy</td>
<td>Deleted</td>
<td>Stored</td>
<td>Deleted</td>
<td>Deleted</td>
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<td>Deleted</td>
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<tr>
<td>5) Cost rate based</td>
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<td>Stored</td>
<td>Deleted</td>
<td>Stored</td>
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<td>Stored</td>
</tr>
<tr>
<td>strategy</td>
<td>(deleted initially)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Local-optimisation based</td>
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<tr>
<td>strategy</td>
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<tr>
<td>7) Minimum cost benchmark</td>
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</table>

### 7.5 Summary

In this Chapter, we demonstrate the experiment results that we conducted on our SwinCloud environment to evaluate the proposed minimum cost benchmarking approaches and cost-effective datasets storage strategies presented in this thesis.

For the minimum cost benchmarking approaches, first, by comparing with some intuitive storage strategies, we demonstrate the cost-effectiveness of the
minimum cost benchmark, and then by comparing the runtime efficiency of the two proposed approaches, i.e. static on-demand approach and dynamic on-the-fly approach, we further demonstrate that two approaches are suitable for different applications with different requirements of benchmarking requests.

For the cost-effective datasets storage strategies, we compare them with the minimum cost benchmark to evaluate their cost-effectiveness. Then we evaluate the efficiency of the strategies by comparing their execution time with the benchmarking approaches. The experiment results indicate that the two proposed strategies have different features, namely, the cost rate based strategy is highly efficient with fairly reasonable cost-effectiveness and the local-optimisation based strategy is highly cost-effective with very reasonable efficiency. They can be utilised in different situations according to the requirements of the applications.

At last, we present the case study conducted on the pulsar searching application in Astrophysics. By utilising our benchmarking approaches and storage strategies in this real world application, we successfully demonstrate the practicability of our research.
Chapter 8

Conclusions and Future Work

In this chapter, we summarise the whole thesis. Section 8.1 summarises the contents of the whole thesis. Section 8.2 outlines the main contributions of this thesis. Finally, Section 8.3 points out the future work.

8.1 Summary of This Thesis

The research objective described in this thesis is to investigate the issue of computation and storage trade-off in the cloud in order to help both users and service provider to bring the cost down dramatically when deploying the computation and data intensive scientific applications with the pay-as-you-go model. The thesis was organised as follows:

- Chapter 1 introduced the scientific applications in the cloud, which is the background of this research. Chapter 1 also described the aims of this work, the key issues to be addressed in this thesis and the primary structure of this thesis.

- Chapter 2 overviewed the related literatures on scientific applications in grid and cloud systems and analysed their limitations. Specifically, first, we overviewed the data management of scientific applications in traditional distributed systems, e.g. grid systems. Next, we reviewed some related work on deploying scientific applications in the cloud and demonstrate the cost-effectiveness of using the cloud. Furthermore, we pointed out that this
research is a step forward based on the existing work, which investigates how to reduce the application cost in the cloud.

- Chapter 3 presented a motivating example of pulsar searching from Astrophysics. Based on the example, we analysed the problems of deploying scientific applications in the cloud and defined the scope of this research. Based on the analysis, we present the detailed research issues of this thesis: 1) cost model for datasets storage in the cloud; 2) minimum cost benchmarking approaches; and 3) cost-effective datasets storage strategies.

- Chapter 4 described a new cost model for datasets storage in the cloud. First we introduced a classification of the application data in the cloud, namely original data and generated data, and proposed the important concept of Data Dependency Graph (DDG). Then, we presented the cost model for datasets storage based on DDG, where the total application cost defined in this thesis is the sum of the computation cost for regenerating datasets and the storage cost for generated datasets. The cost model represents the trade-off between computation and storage, which was investigated in this thesis to reduce the application cost in the cloud.

- Chapter 5 described two novel minimum cost benchmarking approaches. This chapter is the core of this thesis, because benchmarking is to calculate the minimum cost of storing the application datasets in the cloud, which achieves the best trade-off between computation and storage. Most of the important theorems and algorithms were presented in this chapter, based on which we proposed two benchmarking approaches (i.e. static on-demand approach and dynamic on-the-fly approach) according to the different requirements of applications in the cloud.

- Chapter 6 described two innovative cost-effective datasets storage strategies. By utilising the trade-off between computation and storage, two cost-effective datasets storage strategies were designed according to the different requirements of applications in the cloud, i.e. the cost rate based strategy (highly efficient with fairly reasonable cost-effectiveness) and the local-
optimisation based strategy (highly cost-effective with very reasonable efficiency).

- Chapter 7 described the experiments and evaluations of this research. First, general random experiments demonstrated that the minimum cost benchmarking approaches can very well evaluate the cost-effectiveness of storage strategies and the proposed cost-effective storage strategies can also be utilised in different situations according to different application requirements in the cloud. Then, the case study on the specific pulsar searching application further demonstrated the practicability of our benchmarking approaches and storage strategies.

In summary, wrapping up all chapters, we can conclude that with the research results in this thesis, i.e. cost model, benchmarking approaches and storage strategies, the application cost in the cloud can be significantly reduced.

8.2 Key Contributions of This Thesis

The significance of this research is that we have investigated a brand new and niche issue in cloud computing, i.e. the trade-off between computation and storage of data in scientific applications. Because of the wide utilisation of the pay-as-you-go model, application cost becomes an important issue concerned for deploying applications in the cloud. This thesis provided a novel way to reduce the application cost via achieving the best trade-off of computation and storage in the cloud.

In particular, the major contributions of this thesis are:

1. For the first time, the issue of computation and storage trade-off for scientific datasets storage in the cloud is comprehensively and systematically investigated. A brand new cost model for datasets storage is proposed based on a novel concept of Data Dependency Graph (DDG). This cost model represents the trade-off between computation and storage in the application cost.
2. For the first time, a static on-demand minimum cost benchmarking approach is proposed. In this approach, a novel Cost Transitive Tournament Shortest Path (CTT-SP) based algorithm is designed to calculate the theoretical minimum application cost of storing the generated datasets in the cloud. This algorithm solves a seemingly NP-hard problem in polynomial time complexity, i.e. $O(n^9)$.

3. For the first time, a dynamic on-the-fly minimum cost benchmarking approach is proposed. With in-depth investigation of the trade-off between computation and storage, a novel concept of Partitioned Solution Space (PSS) is proposed. Based on PSS, we develop an innovative approach that can dynamically derive the minimum cost benchmark on the fly at runtime in the cloud.

4. For the first time, a cost rate based datasets storage strategy is proposed. This strategy is highly efficient with fairly reasonable cost-effectiveness, which contains three new algorithms to handle all situations (i.e. for new datasets, stored datasets and regenerated datasets) in the cloud to decide the proper storage status of the application datasets.

5. For the first time, a local-optimisation based datasets storage strategy is proposed. This strategy is highly cost-effective with very reasonable efficiency, which contains an enhanced CTT-SP algorithm to decide the proper storage status of the application datasets.

6. A case study is conducted on a real world scientific application, i.e. pulsar seraching in Astrophysics. All proposed benchmarking approaches and storage strategies are utilised in the case study, which demonstrates the practicability of the research outcomes presented in this thesis.

8.3 Future Work

Based on the current work in this thesis, future work can be conducted from the following aspects:
The current work in this thesis is based on Amazon clouds’ cost model and assumes that all the application data be stored with a single cloud service provider. However, sometimes large-scale applications have to run in a more distributed manner since some application data may be distributed with fixed locations. In these cases, data transfer is inevitable. In the future, we will incorporate the data transfer cost into our minimum cost benchmarking and develop more complex cost models. Furthermore, data placement strategies also need to be investigated in order to reduce data transfer among data centres.

The current work in this thesis has an assumption that the datasets’ usage frequencies are obtained from the system log. Models of forecasting dataset usage frequency can be further studied, with which our benchmarking approaches and storage strategies can be adapted more widely to different types of applications.

The datasets storage strategies proposed in this thesis are cost effective and efficient, but not aimed at reaching the minimum cost. Hence more cost-effective storage strategies can be further investigated in order to achieve the minimum cost reflected by benchmarking.
Bibliography


Appendix A
Proofs of Theorems, Lemmas and Corollaries

Theorem 5.1: Given a linear DDG with datasets \{d_1, d_2 \ldots d_n\}, the length of \(P_{\text{min}}<d_s, d_e>\) of its CTT is the minimum cost rate for storing the datasets in the DDG, and the corresponding storage strategy is to store the datasets that \(P_{\text{min}}<d_s, d_e>\) traverses.

Proof of Theorem 5.1:

First, there is a one-to-one mapping between the storage strategies of the DDG and the paths from \(d_s\) to \(d_e\) in the CTT. Given any storage strategy of the DDG, we can find an order of these stored datasets, since the DDG is linear. Then we can find the exact path in the CTT that has traversed all these stored datasets. Similarly, given any path from \(d_s\) to \(d_e\) in the CTT, we can find the datasets it has traversed, which is a storage strategy. Second, based on the setting of weights to the edges, the length of a path from \(d_s\) to \(d_e\) in the CTT equals to the total cost rate of the corresponding storage strategy. Third, \(P_{\text{min}}<d_s, d_e>\) is the shortest path from \(d_s\) to \(d_e\) as found by the Dijkstra algorithm.

Theorem 5.1 holds.

Corollary 5.1: During the process of finding the shortest path, for every dataset \(d_f\) that is discovered by the Dijkstra algorithm, we have a path \(P_{\text{min}}<d_s, d_f>\) from
d_s to d_f and a set of datasets S_f that \( P_{\text{min}} < d_s, d_f > \) traverses. S_f is the MCSS of the sub DDG segment \( \{ d_i \mid d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f \} \).

**Proof of Corollary 5.1:**

Corollary 5.1 is proved by apagoge.

Suppose that there exists a storage strategy \( S_f' \neq S_f \) and \( S_f' \) is the MCSS of the sub-DDG segment \( \{ d_i \mid d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f \} \). Then we can get a path \( P'_{\text{min}} < d_s, d_f > \) from \( d_s \) to \( d_f \), which traverses the datasets in \( S_f' \). Then we have:

\[
P'_{\text{min}} < d_s, d_f > = \left( \sum_{d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f} \text{CostR} \right)_{S_f'}
\]

\[
< \left( \sum_{d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f} \text{CostR} \right)_{S_f} = P_{\text{min}} < d_s, d_f >
\]

This is contradictory to the known condition \( "P_{\text{min}} < d_s, d_f > \) is the shortest path from \( d_s \) to \( d_f" \). Hence, \( S_f \) is the MCSS of the sub-DDG segment \( \{ d_i \mid d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_f \} \).

**Corollary 5.1 holds.**

**Theorem 5.2:** The selection of main branch in the DDG to construct CTT has no impact on finding the MCSS.

**Proof of Theorem 5.2:**

Assume that strategy \( S \) be the MCSS of a DDG; the DDG have two sub-branches \( Br_1 \) and \( Br_2 \) in a block; strategies \( S_1 \) and \( S_2 \) contain the sets of stored datasets of \( Br_1 \) and \( Br_2 \) in \( S \).

If we select the main branch with the sub-branch \( Br_1 \), \( S \) can be mapped to a path in one of the created CTTs. According to Theorem 5.1, the paths in CTT have one-to-one mapping to the storage strategies, hence we can find a path \( P < d_s, d_f > \) that traverses the stored datasets in the main branch according to \( S \). If \( S_1 = \emptyset \), there is an
over-block edge in the path $P<d_s, d_e>$, which contains the MCSS of $Br_2$ according to formula (5.2), where $P<d_s, d_e>$ is in the initial CTT. If $S_i \neq \emptyset$, there is an in-block edge and an out-block edge in $P<d_s, d_e>$, denoted as $e<d_i, d_j>$ and $e<d_h, d_k>$. The weight of $e<d_h, d_k>$ contains the MCSS of $Br_2$ according to formula (5.2), hence $P<d_s, d_e>$ is in CTT($e<d_i, d_j>$). Similar to Theorem 5.1, we can further prove that the length of $P<d_s, d_e>$ equals the total cost rate of the storage strategy $S$.

Similarly, if we select the main branch with the sub-branch $Br_2$, $S$ can also be mapped to a path in one of the created CTTs, where the length of the path equals to the total cost rate of the MCSS.

Therefore, no matter which branch we select as main branch to construct CTT, the MCSS always exists in one of the created CTTs. This means that the selection of main branch has no impact on finding the MCSS.

**Theorem 5.2 holds.**

**Theorem 5.3:** The Dijkstra shortest path algorithm is still applicable to find the MCSS of the DDG with one block.

**Proof of Theorem 5.3:**

In the CTTs created for the DDG with one block, every path from $d_s$ to $d_e$ contains an out-block edge or over-block edge. According to formula (5.2), the minimum cost rate of the sub-branch is contained in the weights of out-block and over-block edges. Hence, every path from $d_s$ to $d_e$ in the CTT contains the MCSS of the sub-branch. Furthermore, the CTTs are created based on the main branch of the DDG, similar to the proof of Theorem 5.1, the shortest path $P_{\min<d_s, d_e>}$ found by the Dijkstra algorithm contains the MCSS of the main branch. This means that $P_{\min<d_s, d_e>}$ represents the MCSS of the whole DDG.

**Theorem 5.3 holds.**
**Theorem 5.4:** For a DDG_LS, only the generation cost of its deleted preceding datasets and the usage frequencies of its deleted succeeding datasets impact on its MCSS.

**Proof of Theorem 5.4:**

We assume that a DDG_LS \( \{d_1, d_2, \ldots, d_n\} \) have \( j \) deleted preceding datasets and \( k \) deleted succeeding datasets, which is shown in Figure A.1.

![A DDG_LS with start and end datasets](image)

**Figure A.1 A DDG_LS with start and end datasets**

In Figure A.1, we can see that the deleted preceding datasets impact on the weights of all the edges from \( d_s \) to the DDG_LS. According to the CTT-SP algorithm, for any dataset \( d_a \) in the DDG_LS, the weight of edge from \( d_s \) to \( d_a \) is

\[
\omega_{d_s, d_a} = y_a + \sum_{d_i \in DDG \land d_s \rightarrow d_i \rightarrow d_a} \left( genCost(d_i) \ast v_i \right)
\]

\[
= y_a + \sum_{i=1}^{j} \left( genCost(d'_i) \ast v'_i \right) + \sum_{i=1}^{a-1} \left( genCost(d_i) \ast v_i \right)
\]

\[
= y_a + \sum_{i=1}^{j} \left( \sum_{h=1}^{i} x'_h \ast v'_i \right) + \sum_{i=1}^{a-1} \left( \sum_{h=1}^{i} x'_h + \sum_{h=1}^{i} x_h \right) \ast v_i
\]

\[
= y_a + \sum_{i=1}^{j} \left( \sum_{h=1}^{i} x'_h \ast v'_i \right) + \sum_{i=1}^{j} x'_h \ast v_j + \sum_{i=1}^{a-1} \left( \sum_{h=1}^{i} x_h \right) \ast v_i
\]

From the composition of \( \omega_{d_s, d_a} \), we can see that

- \( \sum_{i=1}^{j} \left( \sum_{h=1}^{i} x'_h \ast v'_i \right) \) is a fixed value for all the edges starting from \( d_s \) to any datasets in the DDG_LS, because it does not contain variable \( a \). Hence it has no impact on finding the MCSS.
\[ a^{-1} \left( \sum_{i=1}^{a-1} \left( \sum_{h=i}^{i} x_h \right) v_i \right) + y_a \] is a value that is independent of the deleted preceding datasets.

The value of \[ \sum_{i=1}^{a-1} x_h v_i \] depends on both the deleted preceding datasets (i.e. \( \sum_{i=1}^{a-1} x_h \)) and the datasets in the DDG_LS (i.e. \( \sum_{i=1}^{a-1} v_i \)), where \( \sum_{i=1}^{a-1} x_h \) is the generation cost of the deleted preceding datasets.

Hence, we can come to the conclusion that only the generation costs of the deleted preceding datasets impacts on the MCSS of the DDG_LS.

Similarly, for an edge from any datasets \( d_h \) in the DDG_LS pointing to \( d_e \), the weight \( \omega < d_h, d_e > \) is

\[
\omega < d_h, d_e > = y_e + \sum_{i=b+1}^{m} \left( \sum_{h=b+1}^{i} x_h \right) v_i + \sum_{i=1}^{k} x_h v_i + \sum_{i=1}^{k} \left( \sum_{h=i}^{i} x_h \right) v_i.
\]

Therefore, only the usage frequencies of the deleted succeeding datasets, i.e. \( \sum_{i=1}^{k} v_i \), impacts on the MCSS of the DDG_LS.

**Theorem 5.4** holds.

**Theorem 5.5**: Given a DDG_LS \( \{d_1, d_2, \ldots, d_m\} \), \( SCR_{\min} \) is the cost rate of MCSS \( S_{x,v} \) with \( X=0, V=0 \), and \( SCR_{\max} \) is the cost rate of MCSS \( S_{1, n t} \) with \( X>y_1/v_1, V>y_{nt}/x_{nt} \). Then we have \( SCR_{\min} < SCR_{ij} < SCR_{\max} \), where \( SCR_{ij} \) is the cost rate of MCSS \( S_{ij} \) with any given \( X \) and \( V \).

**Proof of Theorem 5.5**:

First, \( SCR_{\min} < SCR_{ij} \) is obviously true because of the direct utilisation of the CTT-SP algorithm. Next, we prove \( SCR_{ij} < SCR_{\max} \) by apagoge.
We assume \( SCR_{i,j} \geq SCR_{\text{max}} \), then we have

\[
TCR_{i,j} = X \times \sum_{k=1}^{i-1} v_k + SCR_{i,j} + V \times \sum_{k=j+1}^{n} x_k
\]

\[
\geq X \times \sum_{k=1}^{i-1} v_k + SCR_{\text{max}} + V \times \sum_{k=j+1}^{n} x_k > SCR_{\text{max}} = TCR_{\text{max}}
\]

This is contradictory to the known condition that \( S_{i,j} \) is the MCSS of the given \( X \) and \( V \).

Theorem 5.5 holds.

Lemmas 5.1 – 5.3 and Theorem 5.6 can be proved in a same way, which is via the linear equation theory in Linear Algebra.

**Lemma 5.1:** In the PSS of a DDG_LS, for three MCSSs, if any two of them are adjacent with each other, then the three partition lines between every two MCSSs intersect at one point.

**Proof of Lemma 5.1:**

For the three lines in Figure 5.15, we can write their equations in the coefficient matrix format, i.e. \( Ax = b \), as follows:

\[
A = \begin{bmatrix}
\sum_{h=j}^{i-1} v_h & \sum_{h=j' + 1}^{i} x_h \\
\sum_{h=k}^{i-1} v_h & \sum_{h=k' + 1}^{i} x_h \\
-\sum_{h=j}^{k-1} v_h & \sum_{h=j' + 1}^{k} x_h
\end{bmatrix}, \quad x = \begin{bmatrix} X \\ V \end{bmatrix}, \quad b = \begin{bmatrix}
(SCR_{j,j'} - SCR_{i,i'}) \\
(SCR_{k,k'} - SCR_{i,i'}) \\
(SCR_{k,k'} - SCR_{j,j'})
\end{bmatrix}
\]

Because of \( d_j \rightarrow d_k \rightarrow d_i \) and \( d_i \rightarrow d_f \rightarrow d_{k'} \), we have \( \sum_{h = j}^{k-1} v_h + \sum_{h = k}^{i-1} v_h = \sum_{h = j}^{i-1} v_h \).
and \( \sum_{h=r+1}^{r'} x_h - \sum_{h=r+1}^{r'} x_h = - \sum_{h=r+1}^{r'} x_h \), hence in matrix \( A \) there are only two linear independent vectors, hence the equation system \( Ax=b \) has a unique solution.

Hence, the three lines (i.e. \( \langle S_{i,i'}, S_{j,j'} \rangle, \langle S_{i,i'}, S_{k,k'} \rangle \) and \( \langle S_{j,j'}, S_{k,k'} \rangle \)) intersect at one point.

**Lemma 5.1 holds.**

**Lemma 5.2:** In a three dimension PSS, for three MCSSs, if any two of them are adjacent with each other, then the three partition planes intersect in one line.

**Proof of Lemma 5.2:**

Similar to the proof of Lemma 5.7, we can write the partition planes’ equations in Figure 5.19 in the coefficient matrix format as follows:

\[
A = \begin{bmatrix}
\sum_{a_1}^{b_1} x & \sum_{a_2}^{b_2} x & \sum_{a_3}^{b_3} x \\
\sum_{b_1}^{c_1} x & \sum_{b_2}^{c_2} x & \sum_{b_3}^{c_3} x \\
\sum_{c_1}^{d_1} x & \sum_{c_2}^{d_2} x & \sum_{c_3}^{d_3} x
\end{bmatrix}, \quad x = \begin{bmatrix}
X_1 \\
V_2 \\
V_3
\end{bmatrix}, \quad b = \begin{bmatrix}
(SCR_b - SCR_a) \\
(SCR_c - SCR_b) \\
(SCR_c - SCR_a)
\end{bmatrix}
\]

Because of \( d_{c_1} \rightarrow d_{b_1} \rightarrow d_{a_1} \), \( d_{a_2} \rightarrow d_{b_2} \rightarrow d_{c_2} \) and \( d_{a_3} \rightarrow d_{b_3} \rightarrow d_{c_3} \), we have \( \sum_{a_1}^{b_1} x + \sum_{b_1}^{c_1} x = \sum_{a_1}^{b_1} x \), \( \sum_{a_2}^{b_2} x + \sum_{b_2}^{c_2} x = \sum_{a_2}^{b_2} x \) and \( \sum_{a_3}^{b_3} x + \sum_{b_3}^{c_3} x = \sum_{a_3}^{b_3} x \), hence in matrix \( A \) there are only two linear independent vectors.

According to the property of 3-variable linear equations, the solution space of the equation system \( Ax=b \) is a line.

Hence, the three lines (i.e. \( P< S_a, S_b >, P< S_b, S_c > \) and \( P< S_a, S_c > \)) intersect in one line.

**Lemma 5.2 holds.**
**Lemma 5.3:** In a three dimension PSS, for four MCSSs, if any three of them intersect in a different line, then the four intersection lines intersect at one point.

**Proof of Lemma 5.3:**

For four MCSSs in a three dimension PSS, the maximum number of linear independent vectors in the partition plane equations’ coefficient matrix is three. We still take Figure 5.19’s DDG segment as example. We assume that $S_e$ be the forth MCSS, where $SCR_a < SCR_b < SCR_c < SCR_e$; $d_{a_1} \rightarrow d_{a_1} \rightarrow d_{b_1} \rightarrow d_{a_1}$, $d_{a_2} \rightarrow d_{b_2} \rightarrow d_{c_2} \rightarrow d_{a_2}$, and $d_{a_3} \rightarrow d_{b_3} \rightarrow d_{c_3} \rightarrow d_{a_3}$. We have partition plane equations of the four MCSSs as follows:

\[
P < S_a, S_b >: \left( \sum_{a_1} b_1 v + \sum_{a_2} b_2 x \right) * X_1 + \left( \sum_{a_3} b_3 \right) * V_2 + \left( \sum_{a_3} c_3 \right) * V_3 = SCR_b - SCR_a
\]

\[
P < S_a, S_c >: \left( \sum_{a_1} c_1 v + \sum_{a_2} c_2 x \right) * X_1 + \left( \sum_{a_3} c_3 \right) * V_2 + \left( \sum_{a_3} c_3 \right) * V_3 = SCR_c - SCR_a
\]

\[
P < S_b, S_c >: \left( \sum_{b_1} c_1 v + \sum_{b_2} c_2 x \right) * X_1 + \left( \sum_{b_3} c_3 \right) * V_2 + \left( \sum_{b_3} c_3 \right) * V_3 = SCR_c - SCR_b
\]

\[
P < S_c, S_c >: \left( \sum_{c_1} c_1 v + \sum_{c_2} c_2 x \right) * X_1 + \left( \sum_{c_3} c_3 \right) * V_2 + \left( \sum_{c_3} c_3 \right) * V_3 = SCR_c - SCR_c
\]

We can clearly see that the linear independent vectors in the equations’ coefficient matrix are $\left[ \sum_{a_1} b_1 v, \sum_{a_2} b_2 x, \sum_{a_3} b_3 x \right], \left[ \sum_{a_1} c_1 v, \sum_{a_2} c_2 x, \sum_{a_3} c_3 x \right], \left[ \sum_{b_1} c_1 v, \sum_{b_2} c_2 x, \sum_{b_3} c_3 x \right], \left[ \sum_{c_1} c_1 v, \sum_{c_2} c_2 x, \sum_{c_3} c_3 x \right]$.

Furthermore, because any three of the four MCSSs intersect in one line, we know that the number of linear independent vectors in the partition plane equations’ coefficient matrix is greater than or equal to two.

If the four MCSSs’ partition plane equations only have two linear independent vectors, then the planes would intersect in the same line according to the
property of linear equations. This is contradictory to the known condition that any three of the four MCSSs intersect in a different line. Hence the four MCSSs’ partition planes’ equations have three linear independent vectors.

According to the property of three variables linear equations, the equation system of the four MCSSs’ partition planes has unique solution. Hence the four MCSSs intersect at one point.

**Lemma 5.3 holds.**

**Theorem 5.6**: In an n dimension PSS, for i MCSSs where \(i \in \{2, 3, \ldots, (n + 1)\}\), if any \((i-1)\) of the \(i\) MCSSs intersect in a different \((n-i+2)\) dimension space, then the \(i\) MCSSs intersect in an \((n-i+1)\) dimension space.

**Proof of Theorem 5.6**

Based on the proofs of Lemma 5.1 – 5.3, Theorem 5.6 can be proved in the same way.

In the \(n\) dimension PSS, the border of two MCSSs is an \(n\)-variable linear equation. For a system of \(n\)-variable linear equations, if its solution is an \(m\) dimension space, then there are \((n-m)\) linear independent vectors in the equations system’s coefficient matrix.

Because any \((i-1)\) of the \(i\) MCSSs intersect in an \((n-i+2)\) dimension space, the \((i-1)\) MCSSs’ equation system has \((i-2)\) linear independent vectors. Furthermore, because different \((i-1)\) MCSSs have different \((n-i+2)\) dimension spaces, the \(i\) MCSSs’ equation system has \((i-1)\) linear independent vectors, which can be proved similarly as Lemma 5.3. Hence the \(i\) MCSSs intersect in an \((n-i+1)\) dimension space.

**Theorem 5.6 holds.**
Theorem 5.7: Given DDG segment \( \{d_1, d_2, \ldots, d_m\} \) with PSS\(_1\), DDG segment \( \{d_{m+1}, \ldots, d_n\} \) with PSS\(_2\), and the merged DDG segment \( \{d_1, d_2, \ldots, d_m, d_{m+1}, \ldots, d_n\} \) with PSS. Then we have:

\[
\forall S \in PSS \Rightarrow S = S_1 \cup S_2, \quad S_1 \in PSS_1 \quad S_2 \in PSS_2
\]

\[
SCR = SCR_1 + \left( \sum_{k=j}^{m} x_k \right) * \left( \sum_{k=m+1}^{i-1} v_k \right) + SCR_2
\]

where \( d_i \) is the last stored dataset in the first DDG segment and \( d_j \) is the first stored dataset in the second DDG segment.

Proof of Theorem 5.7:

As stated in Theorem 5.7, in the merged DDG segment under storage strategy S, the regenerations of datasets in DDG segment \( \{d_{m+1}, d_{m+2}, \ldots, d_{i-1}\} \) need to start from \( d_j \), which includes the generation cost datasets in DDG segment \( \{d_{j+1}, d_{j+2}, \ldots, d_m\} \).

Hence,

\[
SCR = SCR_1 + \left( \sum_{k=j+1}^{m} x_k \right) * \left( \sum_{k=m+1}^{i-1} v_k \right) + SCR_2
\]

can be proved by direct utilisation of the definition of SCR, where

\[
\left( \sum_{k=j+1}^{m} x_k \right) * \left( \sum_{k=m+1}^{i-1} v_k \right)
\]

is the generation cost rate compensation of datasets in DDG segment \( \{d_{j+1}, d_{j+2}, \ldots, d_m\} \) for regenerating datasets in DDG segment \( \{d_{m+1}, \ldots, d_{i-1}\} \).

Next, we prove

\[
\forall S \in PSS \Rightarrow S = S_1 \cup S_2, \quad S_1 \in PSS_1 \quad S_2 \in PSS_2
\]

by apagoge.

We assume \( S_1 \notin PSS_1 \).

Then we write the total cost rate of the merged DDG segment with MCSS S:
\[ TCR = \sum_{h=1}^{p} (X_h \ast \sum_{k} v_k) + SCR + \sum_{h=1}^{q} (V_h \ast \sum_{k} x_k), \]

where \( p \) and \( q \) are the numbers of branches in the merged DDG segment that have preceding datasets and succeeding datasets. Then we have

\[ TCR = \sum_{h=1}^{p} (X_h \ast \sum_{k} v_k) + SCR + \sum_{h=1}^{q} (V_h \ast \sum_{k} x_k) \]

\[ = \sum_{h=1}^{p} (X_h \ast \sum_{k} v_k) + SCR_1 + \left( \sum_{k=j+1}^{m} x_k \right) \ast \left( \sum_{k=m+1}^{i-1} v_k \right) + SCR_2 + \sum_{h=1}^{q} (V_h \ast \sum_{k} x_k) \]

\[ = \sum_{h=1}^{p_1} (X_h \ast \sum_{k} v_k) + SCR_1 + \sum_{h=1}^{q_1} (V_h \ast \sum_{k} x_k) + \left( \sum_{k=j+1}^{m} x_k \right) \ast \left( \sum_{k=m+1}^{i-1} v_k \right) \]

\[ + \sum_{h=1}^{p_1} (X_h \ast \sum_{k} v_k) + SCR_2 + \sum_{h=1}^{q_1} (V_h \ast \sum_{k} x_k) \]

where \( p_1 \) and \( q_1 \) are the numbers of branches in the DDG segment \( \{d_1, d_2, \ldots, d_m\} \) that have preceding datasets and succeeding datasets except the connecting branch; \( p_2 \) and \( q_2 \) are the numbers of branches in the DDG segment \( \{d_{m+1}, d_{m+2}, \ldots, d_n\} \) that have preceding datasets and succeeding datasets except the connecting branch. Next, we have

\[ TCR = TCR_1 + \sum_{h=1}^{p_1} (X_h \ast \sum_{k} v_k) + SCR_2 + \sum_{h=1}^{q_1} (V_h \ast \sum_{k} x_k) \]

Since \( S_1 \notin PSS_1 \), given the \( X \) values \( \{X_1, X_2, \ldots, X_{p_1}\} \), \( V \) values \( \{V_1, V_2, \ldots, V_{q_1}\} \)

and \( V = \sum_{k=m+1}^{i-1} v_k \), we can find another MCSS \( S'_1 \), where \( TCR_1' < TCR_1 \). Hence, we have
\[ TCR = TCR_i + \sum_{h=1}^{p_i} (X_h \times \sum_v v_k) + SCR_2 + \sum_{h=1}^{q_i} (V_h \times \sum x_k) \]

\[ > TCR'_i + \sum_{h=1}^{p_i} (X_h \times \sum_v v_k) + SCR_2 + \sum_{h=1}^{q_i} (V_h \times \sum x_k) \]

\[ = \sum_{h=1}^{p_i} (X_h \times \sum_v v'_k) + SCR'_i + \sum_{h=1}^{q_i} (V_h \times \sum x'_k) + \left( \sum_{k=j+1}^{m} x_k \right) * \left( \sum_{k=m+1}^{i-1} v_k \right) \]

\[ + \sum_{h=1}^{p_i} (X_h \times \sum_v v_k) + SCR_2 + \sum_{h=1}^{q_i} (V_h \times \sum x_k) \]

\[ = \sum_{h=1}^{p_i} (X_h \times \sum_v v'_k) + SCR' + \sum_{h=1}^{q_i} (V_h \times \sum x_k) = TCR' \]

This is contradictory to the known condition that \( S \) is the MCSS of the merged DDG Segment.

Hence \( S_1 \in PSS_1 \).

Similarly, we can prove \( S_2 \in PSS_2 \).

**Theorem 5.7 holds.**

**Lemma 6.1:** *The deletion of a stored dataset in the DDG does not affect the storage status of other stored datasets.*

**Proof of Lemma 6.1:**

Suppose that \( d_i \) be a stored datasets to be deleted, \( d_p \) be a stored predecessor of \( d_i \) and \( d_j \) be a stored successor of \( d_i \). If \( d_i \) is deleted, 1) more datasets’ regenerations need to use \( d_p \), i.e. the deleted successors of \( d_i \), hence \( d_p \) still needs to be stored; 2) the regeneration of \( d_j \) needs to start from \( d_p \) and regenerate the deleted predecessors of \( d_i \), hence the generation cost of \( d_j \) is increased and \( d_j \) still needs to be stored.

**Lemma 6.1 holds.**
Theorem 6.1: If a deleted dataset is stored, only its adjacent stored predecessors and successors in the DDG may need to be deleted to reduce the application cost.

Proof of Theorem 6.1:

Suppose that \( d_i \) be a deleted datasets to be stored, \( d_p \) be a stored predecessor of \( d_i \) and \( d_f \) be a stored successor of \( d_i \). If \( d_i \) is stored, 1) fewer datasets’ regenerations need to use \( d_p \), i.e. regenerations of the deleted successors of \( d_i \) can start from \( d_i \), hence \( d_p \) may needs to be deleted; 2) the regeneration of \( d_f \) can start from \( d_i \) instead of \( d_p \), hence the generation cost of \( d_f \) is decreased and \( d_f \) may need to be deleted. According to Lemma 6.1, the deletion of \( d_p \) and \( d_f \) do not affect other stored datasets’ storage status.

Theorem 6.1 holds.

Theorem 6.2: Given a DDG and assumed \( S \) be the MCSS of the DDG. If \( d_p \in S \) and \( d_p \) divides the DDG into:

\[
\begin{align*}
\text{DDG}_1 = \{ d_j | d_j \in \text{DDG} \land d_j \rightarrow d_p \} \\
\text{DDG}_2 = \{ d_k | d_k \in \text{DDG} \land d_p \rightarrow d_k \}
\end{align*}
\]

then \( S_1 \) and \( S_2 \) are the MCSSs of \( \text{DDG}_1 \) and \( \text{DDG}_2 \) respectively, where \( S_1 = S \cap \text{DDG}_1 \) and \( S_2 = S \cap \text{DDG}_2 \).

Proof of Theorem 6.2:

We prove this theorem by apagoge.

1) Suppose there be a storage strategy \( S'_1 \neq S_1 \) and \( S'_1 \) be the MCSS of \( \text{DDG}_1 \). Then we have:
\[
\left( \sum_{d \in \text{DDG}_1} \text{Cost}_{R_i} \right)_{S_1} < \left( \sum_{d \in \text{DDG}_1} \text{Cost}_{R_i} \right)_{S_1} \\
\Rightarrow \left( \sum_{d \in \text{DDG}_1} \text{Cost}_{R_i} \right)_{S_1} + y_p + \left( \sum_{d \in \text{DDG}_2} \text{Cost}_{R_i} \right)_{S_2} \\
< \left( \sum_{d \in \text{DDG}_1} \text{Cost}_{R_i} \right)_{S_1} + y_p + \left( \sum_{d \in \text{DDG}_2} \text{Cost}_{R_i} \right)_{S_2} \\
\Rightarrow \left( \sum_{d \in \text{DDG}_1} \text{Cost}_{R_i} \right)_{S_1} + y_p + \left( \sum_{d \in \text{DDG}_2} \text{Cost}_{R_i} \right)_{S_2} < \left( \sum_{d \in \text{DDG}} \text{Cost}_{R_i} \right)_{S}
\]

Then \( \left( \sum_{d \in \text{DDG}} \text{Cost}_{R_i} \right)_{S'} < \left( \sum_{d \in \text{DDG}} \text{Cost}_{R_i} \right)_{S}, \ S' = S_1 \cup \{d_p\} \cup S_2. \)

Hence we get a new storage strategy \( S' \) of the DDG which has a smaller cost rate than \( S \). This is contradicting to the known condition “\( S \) is the MCSS of the DDG”. Hence \( S_1 \) is the MCSS of DDG\(_1\).

2) Similarly, it can be proved that \( S_2 \) is the MCSS of DDG\(_2\).

**Theorem 6.2 holds.**
## Appendix B

### Notation Index

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<tr>
<td>$P\langle S_1, S_2\rangle$</td>
<td>Partition plane between MCSS $S_1$ and $S_2$ in a three dimension PSS</td>
</tr>
<tr>
<td>$P_{\min}\langle d_i, d_j\rangle$</td>
<td>The shortest path from $d_i$ to $d_j$ in the CTT</td>
</tr>
<tr>
<td>$Price_{cpu}$</td>
<td>The price of computation resources in the cloud</td>
</tr>
<tr>
<td>$provSet_i$</td>
<td>Set of stored provenance datasets that are needed for regenerating $d_i$</td>
</tr>
<tr>
<td>PSS</td>
<td>Partitioned Solution Space</td>
</tr>
<tr>
<td>$S$</td>
<td>A storage strategy which is a set of datasets in the corresponding DDG (or DDG segment)</td>
</tr>
<tr>
<td>$S_i$</td>
<td>A storage strategy of a DDG (or DDG segment), where the subscript $i$ is the index number</td>
</tr>
<tr>
<td>$S_{i,...,j}$</td>
<td>A storage strategy, where the subscripts $i,...,j$ denote the indices of the first and last stored datasets in the DDG segment</td>
</tr>
<tr>
<td>$S_{\max}$</td>
<td>The MCSS that has the maximum SCR in the PSS</td>
</tr>
<tr>
<td>$S_{\min}$</td>
<td>The MCSS that has the minimum SCR in the PSS</td>
</tr>
<tr>
<td>$S_{All}$</td>
<td>Set of MCSSs of a DDG segment with SCR values in the valid range</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>SB</td>
<td>Sub-branch(es) of a DDG</td>
</tr>
<tr>
<td>$S_{ini}$</td>
<td>Set of MCSSs for the initial input of calculating PSS</td>
</tr>
<tr>
<td>SCR</td>
<td>Sum of cost rates of datasets in a DDG (or DDG segment)</td>
</tr>
<tr>
<td>$SCR_i$</td>
<td>The SCR with storage strategy $S_i$</td>
</tr>
<tr>
<td>$SCR_{i,...,j}$</td>
<td>The SCR with storage strategy $S_{i,...,j}$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>The time duration which denotes user’s tolerance of dataset accessing $d_i$’s delay</td>
</tr>
<tr>
<td>TCR</td>
<td>Total Cost Rate of a DDG segment in the whole DDG</td>
</tr>
<tr>
<td>$TCR_i$</td>
<td>The TCR with storage strategy $S_i$</td>
</tr>
<tr>
<td>$TCR_{i,...,j}$</td>
<td>The TCR with storage strategy $S_{i,...,j}$</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Usage frequency of dataset $d_i$</td>
</tr>
<tr>
<td>$V$</td>
<td>Sum of deleted succeeding datasets usage frequencies of a DDG_LS</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Generation cost of dataset $d_i$ from its direct predecessors</td>
</tr>
<tr>
<td>$X$</td>
<td>Sum of deleted preceding datasets generation costs of a DDG_LS</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Storage cost rate dataset $d_i$</td>
</tr>
<tr>
<td>$\omega &lt; d_i, d_j &gt;$</td>
<td>The weight of edge $e &lt; d_i, d_j &gt;$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>User’s preference of storing dataset $d_i$ with a higher storage cost</td>
</tr>
<tr>
<td></td>
<td>Denotation of two datasets having a generation relationship</td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>→</td>
<td>Denotation of two datasets not having a generation relationship</td>
</tr>
</tbody>
</table>

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