

A semi-closed-form solution for chloride diffusion in concrete with time-varying parameters

T. Maheswaran* and J. G. Sanjayan*

Monash University, Australia

Reinforcement corrosion in concrete is a worldwide problem and billions of dollars are spent each year on repair and replacement of reinforced concrete structures because of corrosion damage of reinforcement. Chloride induced corrosion is the major type of corrosion encountered in reinforced concrete. The corrosion is initiated in reinforcement when chloride diffuses through the concrete cover and the chloride concentration at the reinforcement reaches a critical threshold level. Fick's second law of diffusion is commonly used for one-dimensional chloride diffusion. A closed-form solution based on error function is commonly used to predict the chloride concentrations, if the diffusion coefficient (D) and surface chloride concentration (C_s) remain constant. This solution is not applicable when D and C_s vary with time, which is the case in real-life situations. The finite difference or finite element methods with specialised software are generally used to solve such problems. This paper reviews existing methods and presents an alternative method based on a general closed-form solution with varying D but constant C_s . To incorporate the effect of varying C_s , an adaptation of the closed-form solution is presented. This solution can be used without the use of finite difference method or special computer software to predict the chloride concentrations.

Notation

C	concentration of diffusing substance	D_{PC}	diffusion coefficient for Portland cement concrete ⁷
C_s	surface chloride concentration (varying)	D_{ref}	diffusion coefficient at time t_{ref}
C_0	constant surface chloride concentration	D_{SF}	diffusion coefficient for concrete with silica fume ⁷
C_t	threshold chloride concentration	$D(t)$	diffusion coefficient at time t
$C(0,t)$	surface chloride concentration ⁶	$D_{Ultimate}$	diffusion coefficient from laboratory tests ¹²
$C(x,t)$	chloride concentration at depth x , at time t	erf	error function
D	diffusion coefficient	F	rate of transfer per unit area of substance
D_a	apparent diffusion coefficient proposed in this paper	K_D	constant governing the influence of dispersion ¹²
D_c	diffusion coefficient ¹²	m	constant
D_{ca}	apparent diffusion coefficient ⁵	n	ageing factor ⁵
$D_{ca(tm)}$	apparent diffusion coefficient measured at time t_m ⁵	T_{C_0}	a constant governing the rate of build-up ¹²
D_i	diffusion coefficient at time equal to 1 s ¹³	T_{Dc}	constant governing the rate of reduction of D_c ¹²
		t	time of chloride exposure
		t_i	age at exposure to chloride calculated as per equations (22) and (24)
		t_{ref}	reference time
		x	space coordinate measured normal to the section

* Department of Civil Engineering, Monash University, Vic 3800, Australia.

(MCR 31194) Paper received 28 October 2003; last revised 8 March 2004; accepted 1 April 2004

Y_b	time for maximum chloride concentration build-up
α	constant ⁶
τ_i	time calculate as per equations (21) and (24)
τ	ratio, (t_{ref}/t)

Introduction

Reinforcement corrosion in concrete is a worldwide problem and billions of dollars are spent each year on repair and replacement because of corrosion damage. Estimates, based on 1975 Battelle-NBS bench mark study, are that cost of corrosion in the United States alone was approximately US\$ 70 billion, which was 4.2% of the gross national product (GNP). In 1995, a limited study updating the 1975 figures estimated the total cost of corrosion at approximately US\$ 300 billion.¹ A significant component of this cost is due to the reinforcement corrosion in concrete. Damage caused by steel corrosion in concrete can be summarised as: (a) rust staining; (b) delamination; (c) cracking; (d) spalling; (e) loss of serviceability; and (f) ultimate failure of the structure.

Reinforcement corrosion in concrete is an electrochemical process. The risk of corrosion is minimal in a well-designed reinforced structure in which the concrete cover provides the physical barrier. The high alkalinity of concrete pore water provides the chemical barrier by forming a passive layer on the steel surface.² The corrosion of steel may begin when the chemical barrier ceases to be effective and the passivating film becomes unstable. This 'depassivation' can occur by the effect of chloride ions that can induce corrosion. Chloride ions act as catalysts to corrosion when there is sufficient concentration (above 0.4% of cement content) at the reinforcing bar surface to breakdown the passive layer. The time taken for the surface chloride to diffuse through the concrete cover and build up the critical concentration at the reinforcement bar level to cause corrosion is termed the initiation period. The estimation of this initiation period is important in the estimation of the service life of the structure. The initiation time depends on various factors including surface chloride concentration, diffusion coefficient and crack widths. Rodriguez and Hooton³ studied the effect of cracks in the chloride diffusion process and concluded that transecting, parallel wall cracks of widths 0.1 to 0.68 mm behave like free concrete surfaces greatly contributing to lateral chloride diffusion. This implies that the diffusion process becomes a case of two-dimensional diffusion. The effects of crack width are not included in this paper, as further studies are required to fully understand the diffusion process in this instance covering all range of crack widths.

Fick's laws

Fick's first law of diffusion illustrates the mathema-

tical theory of diffusion in isotropic substances.⁴ This is based on the hypothesis that the rate of transfer of diffusing substance through unit area of a section is proportional to the concentration gradient measured normal to the section,

$$F = -D \frac{\partial C}{\partial x} \quad (1)$$

where F is the rate of transfer per unit area of section; C is the concentration of the diffusing substance; x is the space coordinate measured normal to the section; and D is the diffusion coefficient.

Fick's second law of diffusion deals with one-dimensional diffusion; if the gradient of concentration is only along the x -axis, equation (1) becomes

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (2)$$

For one-dimensional flow into a semi-infinite medium a closed-form solution can be derived for equation (2) as follows

$$C(x, t) = C_0 \left(1 - \operatorname{erf} \frac{x}{2\sqrt{(Dt)}} \right) \quad (3)$$

where $C(x, t)$ is the concentration at depth x (m), at time t (% by mass); t is the time of exposure (s); C_0 is the surface chloride concentration (% by mass); D is the constant diffusion coefficient (m^2/s); and erf is the error function.

Equation (3) can be used only if the diffusion coefficient, D , and surface chloride concentration, C_s , remain constant during the time of diffusion. It has been shown by previous researchers⁵⁻⁷ that these conditions do not apply in practical conditions.

The quality of concrete improves with time and the diffusion coefficient, D , is a function of permeability and decreases with time. This decrease of D with time depends on the type of binder used, total binder content and water/cement ratio. The effects of slag and fly ash on the early-age diffusion coefficient, D_{28} , are inconclusive, but they do significantly influence the time-dependent nature of D .⁵ This provides an explanation for the superior long-term durability of concrete made with slag-blended cement concrete. Use of silica fume in concrete causes a significant reduction of D at early age,^{8,9} however the reduction of D over time is not as significant as with the case of slag. The chemical corrosion inhibitors reduce the value of D_{28} , modify the chloride threshold value, C_t and change the rate of chloride build-up on the surface of the concrete.¹⁰ There are many products available in the market and the effects depend on the dosage levels and type of inhibitors.

Equation (2) can be modified to take account of the variation of D with time as follows⁴

$$\frac{\partial C}{\partial t} = D(t) \frac{\partial^2 C}{\partial x^2} \quad (4)$$

The finite difference method is generally used to solve

equation (4). A closed-form solution to this differential equation including the time-varying surface chloride concentration and diffusion coefficient has not been presented in the literature. This paper presents a general closed-form solution to equation (4) including diffusion coefficient variation with time while surface chloride concentration remains constant. The solution is then adapted to include the time-varying surface chloride concentration. This solution method can be used without the use of the finite difference method or special computer software to predict the chloride concentrations.

Current methods for prediction

Life-365 Version 1.0

Life-365,⁷ a computer program for predicting the service life and life-cycle costs of reinforced concrete exposed to chlorides was developed by a consortium established under the Strategic Development Council of the American Concrete Institute. This software presents an initial life cycle cost model based on an existing service life model developed at the University of Toronto.¹¹ The version 1.0 represents the first phase of a long-term goal to develop a comprehensive service life and life cycle model for reinforced concrete. The current version has many limitations in that a number of assumptions or simplifications have been made to deal with some of the more complex phenomena or areas where there is insufficient knowledge to permit a more rigorous analysis.

Life-365 software uses the finite difference method to solve equation (4). The following relationship is used to account for time-dependent changes in D

$$D(t) = D_{\text{ref}} \left(\frac{t_{\text{ref}}}{t} \right)^m \quad (5)$$

where $D(t)$ is the diffusion coefficient at time t ; D_{ref} is the diffusion coefficient at some reference time t_{ref} (= 28 days); and m is a constant (depending on concrete mix proportions).

The following values are assumed in this software for Portland cement concrete with no special corrosion protection applied

$$D_{\text{PC}} = 1 \times 10^{(-12.06+2.4w/c)} \quad (6)$$

$m = 0.2$ and $C_t = 0.05\%$ (by mass of concrete), where C_t = threshold chloride content for corrosion initiation.

Life-365 applies a reduction factor to the D value calculated for PC concrete based on the level of silica fume (%SF) in the concrete. This relationship is only valid up to replacement levels of 15% silica fume. The model will not compute diffusion values (or make service life predictions) for higher levels of silica fume.

$$D_{\text{SF}} = D_{\text{PC}} e^{-0.165\text{SF}} \quad (7)$$

Life-365 assumes that silica fume has no effect on either m or C_t .

Neither fly ash nor slag is assumed in Life-365 to affect the early-age diffusion coefficient, D_{28} , or C_t . However, the rate of reduction in diffusivity is affected, hence the value of m . The following equation is used in Life-365 to modify the value of m based on the level of fly ash (%FA) or slag (%SG) in the mix.

$$m = 0.2 + 0.4(\%FA/50 + \%SG/70) \quad (8)$$

This relationship is only valid up to replacement levels of 50% fly ash or 70% slag, and thus $m \leq 0.6$. The model will not compute diffusion values (or make service life predictions) for higher levels of these materials.

Life-365 accounts for two, calcium nitrite (CNI) and Rheocrete 222+, chemical corrosion inhibitors at this time. The authors of Life-365 intend to incorporate more corrosion inhibitors when appropriate documentation on the performance becomes available. The membranes and sealers are assumed to impact the rate of chloride build-up only. It is assumed in the model that the rate of damage build up is lower when epoxy-coated steel is present and these effects are dealt by modifying the propagation period, t_p . The initiation period, t_i , is not influenced by the presence of epoxy-coated steel. The chloride threshold of $C_t = 0.5\%$ (by mass of concrete) is assumed for grade 316 stainless steel in the current version of Life-365.

There are limitations in using this software: (a) this software needs to be used as a black-box when using it for prediction of chloride profiles; (b) the user has limited influence on the way the calculations are performed and cannot modify the model; and (c) it would not be possible for the user to incorporate this software as a part of another analysis, as the results from this software have to be fed into other analyses manually. For example, Monte Carlo simulation of the diffusion processes, typically carried out to estimate the reliability of the service life of a structure would be almost impossible to perform.

Bamforth's method

Bamforth's⁵ research indicated very little time-dependent change in diffusion coefficient for the PC concrete and the silica fume mix, but significant reduction for the pulverised-fuel ash (PFA) and ground granulated blastfurnace slag (GGBS) mixes. These trends were further confirmed by review of published data for various concrete mixes. The following relationship to take account of the time-dependent diffusion coefficient was proposed by Bamforth⁵ after analysing the data.

$$D_{\text{ca}} = at^n \quad (9)$$

The coefficient a is the value of D_{ca} at time $t = 1$ year, and n is the slope of the line relating to $\log D_{\text{ca}}$ and $\log t$. The coefficients a and n are derived from regression analysis of the data.

Bamforth⁵ has modified the error function solution to take account of ageing as follows:

$$C(x, t) = C_0 \left(1 - \operatorname{erf} \frac{x}{2\sqrt{[D_{ca(t_m)}(t/t_m)^n t]}} \right) \quad (10)$$

where $D_{ca(t_m)}$ is the apparent diffusion coefficient measured at time t_m ; and n is the ageing factor.

Limitations in using this model are: (a) $D_{ca(t_m)}$ is an approximation of time-dependent D and does not follow Fick's law; (b) the variation in surface chloride is not considered in this model. The validity of using apparent D to take into account the reduction of D over time is tested in detail below.

Collins and Grace's method

Collins and Grace¹² developed a computer model which allows both D and surface chloride concentration C_s to be varied with time. The model was developed from the real in-situ chloride penetration measured in a jetty structure in Hong Kong monitored over a 7-year period. By calibrating the model against the actual measured chloride profiles in the structure, the algorithms that are believed suitable for more general application were determined. The relationship chosen to represent the variation in C_s was

$$C_s = C_0 \frac{t}{t + T_{C_0}} \quad (11)$$

where t is the time of exposure in days and T_{C_0} is a constant governing the rate of build-up.

The best fit for the empirical data gave a value of $T_{C_0} = 500$ in the case of the Collins and Grace example. A different rate of build-up can be accommodated for different concretes and/or different exposure conditions by varying this factor.

The effects of moisture movement at an early age in unsaturated concrete can result in much more rapid chloride penetration in the cover concrete than due to diffusion only. However once a concentration is established in this zone, the relative influences of moisture movement reduce, and ionic diffusion increase. The following algorithm was developed by Collins and Grace¹² to accommodate these variations due to the iteration of the data.

$$D_c = (K_D - 1)D_{Ultimate} \left(1 - \frac{t}{t + T_{D_c}} \right) + 1 \quad (12)$$

where K_D is a constant governing the influence of dispersion; and T_{D_c} is a constant governing the rate of reduction of D_c .

The best fit to the empirical data in the sample case was $K_D = 2$ and $T_{D_c} = 100$, taking $D_{Ultimate} = 3 \times 10^{-12} \text{ m}^2/\text{s}$ from the laboratory diffusion test.

A computer-based finite difference numerical model was used to allow these parameters to vary with time in chloride concentration predictions.

Mangat and Molloy's method

Mangat and Molloy's¹³ research using different concrete mixes for exposure period of up to 5 years under tidal cycles in the North Sea and marine cycles in the laboratory indicated the trend of decreasing diffusion coefficient with time and the rate of decrease being sharp at early ages. Mangat and Molloy¹³ proposed that the time dependence of D might be approximated by the empirical relationships of the form.

$$D_c = D_i t^{-m} \quad (13)$$

where D_i is the diffusion coefficient at time equal to 1 s; and m is the empirical coefficient influenced by w/c ratio.

Mangat and Molloy¹³ developed a closed-form solution incorporating the relationship in equation (13) to account for time-dependent diffusion process as follows

$$C(x, t) = C_0 \left(1 - \operatorname{erf} \frac{x}{2\sqrt{\frac{D_i}{1-m} t^{1-m}}} \right) \quad (14)$$

The variation in surface chloride concentration is not incorporated in this model. It is also worth noting that the equation (14) is a special case of the general closed-form solution derived in the equation (20) and can be obtained by substituting $t_i = t_{ref} = 1$. Mangat and Molloy¹³ carried out the integration for T from $t = 0$ which leads to a diffusion coefficient of infinity at $t = 0$.

Kassir and Ghosn's method

Kassir and Ghosn's⁶ study on published data indicates that the surface chloride concentration varies with time and constant chloride accumulation at concrete surface commonly used is not accurate. The regression analysis of the field data performed by them produced an exponential equation for the surface chloride as follows

$$C(0, t) = C_0(1 - e^{-\alpha t}) \quad (15)$$

where $\alpha = 0.25/\text{year}$.

Kassir and Ghosn⁶ developed a closed-form solution to predict the chloride concentration and corrosion initiation time where surface chloride build-up was exponentially varied with time as in equation (15). The diffusion coefficient (D_0) was taken as constant during exposure and the solution taking account of exponential surface chloride build-up is as follows

$$C(x, t) = C_0 \left[1 - \operatorname{erf} \left(\frac{x}{\sqrt{4D_0 t}} \right) - e^{-x^2/4D_0 t} \operatorname{Re} \left(e^{-z^2} \operatorname{erfc}(-iz) \right) \right] \quad (16)$$

where $i = \sqrt{-1}$ is the usual complex number; $\operatorname{Re}(\)$ stands for the real part of function(), and

$$z = \sqrt{at} + i \frac{x}{\sqrt{4D_0t}}$$

The variation in D with time is not incorporated in this model.

Proposed method of solution

Closed-form solution for time-varying D

A general closed-form solution to the equation (4) can be developed by modifying the equation as follows⁴

$$dT = D(t)dt \tag{17}$$

and
$$T = \int_{t_i}^t D(t')dt' \tag{18}$$

Then equation (4) reduces to

$$\frac{\partial C}{\partial T} = \frac{\partial^2 C}{\partial x^2} \tag{19}$$

Equation (19) is solved below based on following assumptions: (a) variation of D with time is of the form presented in equation (5); and (b) the surface chloride concentration is constant with time. The solution to equation (19) can be found to be as in equation (20)

$$C(x, t) = C_0 \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{\frac{D_{\text{ref}}(t_{\text{ref}})^m}{1-m}((t)^{1-m} - (t_i)^{1-m})}} \right) \right] \tag{20}$$

where D_{ref} is the diffusion coefficient measured at time t_{ref} ; t_i is the age at first exposure to chloride; C_0 is the constant surface chloride concentration and m is a constant (depending on concrete proportions).

Adaptation of the closed-form solution to time-varying C_s

A time-varying C_s and D model is a major advance over the constant C_0 model. Equation (20) is derived for constant C_0 , but different times for first exposure to chloride can be used in this equation. Therefore, this equation can be adapted by assuming the variation of C_s occurs in steps and the principle of superposition can be used to get the total chloride concentration. The variation of C_s is assumed to be linear (Life-365⁷) in some cases whereas this variation is assumed to be exponential in other cases.⁶ The type of function used to describe the variation does not make a significant difference in the long-term results.

Linear surface-chloride build-up and time-dependent diffusion coefficient

Figure 1 shows a linear surface-chloride build-up where the age at first exposure to chloride is t_{ref} . The maximum chloride C_0 is reached at time Y_b . The linear-surface chloride build-up can be approximated to step-

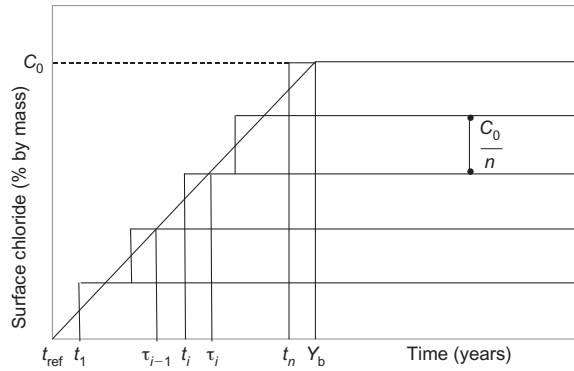


Fig. 1. Linear surface-chloride build-up

by-step deposits of chloride on the concrete surface. The linear increase in surface-chloride concentration can be approximated into n equal numbers of deposits each with a chloride concentration of C_0/n on the concrete surface at times $t_1, t_2 \dots t_n$. The times $t_1, t_2 \dots t_n$ are calculated to ensure that there is no loss of chloride concentration due to this approximation. Equation (20) can be used to calculate the diffusion of chloride due to each step deposit of chloride. The equation for the prediction of total chloride concentration can be derived by the summation of the diffusion of each step as shown in equation (23)

$$\tau_i = \frac{i}{n}(Y_b - t_{\text{ref}}) + t_{\text{ref}} \tag{21}$$

$$t_i = \frac{\tau_{i-1} + \tau_i}{2} \tag{22}$$

where $\tau_0 = t_{\text{ref}}$, and

$$C(x, t) = \sum_{i=1}^n \frac{C_0}{n} \times \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{\frac{D_{\text{ref}}}{(1-m)}(t_{\text{ref}})^m [(t)^{1-m} - (t_i)^{1-m}]}} \right) \right] \tag{23}$$

Exponential surface-chloride build-up and time-dependent diffusion coefficient

Figure 2 shows an exponential surface-chloride build-up where the chloride build-up is described by an exponential equation, $C_s = C_0(1 - e^{-\alpha(t-t_{\text{ref}})})$. The exponential curve can also be approximated by a number of steps similar to the linear one described in the preceding section. The only difference is that the times of the steps $t_1, t_2, \dots t_n$ are equally spaced for the linear case whereas in exponential build-up they are not equally spaced. The time steps are chosen such that area under the curve lost due to the step function is equal to the area gained by the step function ($A_1 = A_2$) as shown in Fig. 3.

The time steps can be calculated from the following equations:

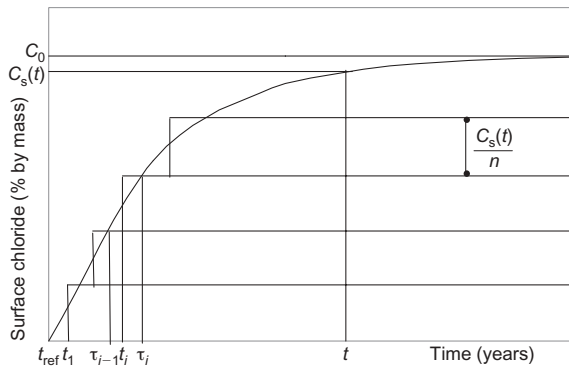


Fig. 2. Exponential surface-chloride build-up

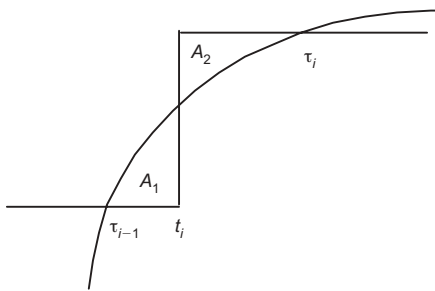


Fig. 3. Selection of t_i ($A_1 = A_2$)

$$\tau_i = t_{ref} - \frac{1}{\alpha} \log_e \left[1 - \frac{iC_s(t)}{nC_0} \right] \quad (24)$$

$$t_i = \frac{n(\tau_{i-1} - \tau_i) + \frac{n}{\alpha} (e^{-\alpha(\tau_{i-1} - t_{ref})} - e^{-\alpha(\tau_i - t_{ref})})}{(1 - e^{-\alpha(t - t_{ref})})} - (i - 1)\tau_{i-1} + i\tau_i \quad (25)$$

where $\tau_0 = t_{ref}$ and $\tau_n = t$.

Once the time steps are calculated, the rest of the calculations are similar as discussed in the preceding section. Therefore, equation (23) can be used to calculate the total chloride concentration.

Apparent diffusion coefficient

The implication of decreasing D with time is that the predictions of long-term chloride concentrations in concrete structures by simply using D values measured at early ages will result in an overestimation of those concentrations. Therefore, incorporating the time dependence of D in the chloride diffusion model used for calculating long-term chloride concentrations should lead to greater accuracy. Bamforth⁵ and Mangat and Molloy¹³ have modelled the decreasing D with time in two different ways as shown in the descriptions of their methods above. It was shown above that Mangat and Molloy's¹³ equation is a special case of equation (20) derived above. The apparent diffusion coefficient pro-

posed by Bamforth⁵ to incorporate the time-varying effect on D is

$$D_{ca} = D_{ref} \left(\frac{t}{t_{ref}} \right)^n \quad (26)$$

The apparent diffusion coefficient proposed in this paper to incorporate the time-varying effect on D assuming that all chloride was exposed to the concrete at t_{ref} is

$$D_a = \frac{D_{ref}}{1 - m} \left[\left(\frac{t_{ref}}{t} \right)^m - \frac{t_{ref}}{t} \right] \quad (27)$$

Equating the apparent diffusion coefficient from equations (26) and (27) gives the relationship between m and n

$$n = \frac{\log_e \left(\frac{\tau^m - \tau}{1 - m} \right)}{\log_e(1/\tau)} \quad (28)$$

where $\tau = t_{ref}/t$

Equation (28) implies that n is a function of t and a constant value of n can not be used in equation (10) for a particular type of concrete as recommended by Bamforth.⁵ Fig. 4 shows the variation of n with time for $m = 0.5$ and $m = 0.8$. The variation of n with time for various m can be plotted using equation (28).

Examples

Linear chloride build-up

Table 1 presents the predicted chloride concentration at various depths for a concrete slab with $w/c = 0.4$.

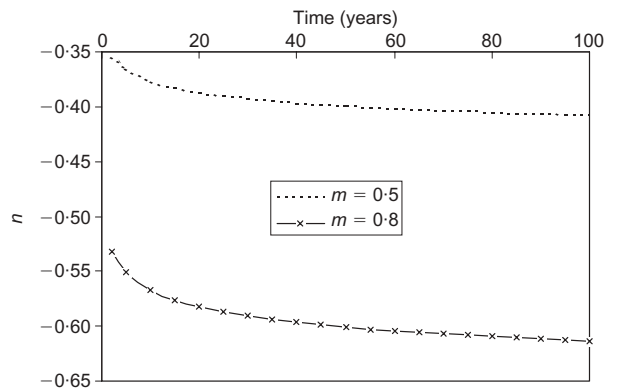


Fig. 4. Variation of n with time

Table 1. Chloride concentration for linear chloride build-up

x (mm)	Prediction by adapted method	Finite difference
10	0.83	0.83
20	0.67	0.67
30	0.52	0.52
40	0.39	0.39
50	0.28	0.28
60	0.20	0.20

The maximum surface chloride (C_0) was equal to 1.0% by mass of concrete. It was assumed that the surface chloride build-up period, (Y_b) was equal to 4 years and the first exposure to chloride occurred at 28 days. The diffusion coefficient ($D_{ref} = 7.94 \times 10^{12}$) was calculated using the equation suggested in Life-365⁷ and m was taken as 0.3. Table 1 values calculated after 20 years of exposure to chloride and five equal steps ($n = 5$) used in this calculation. It can be seen from Table 1 that the predicted chloride concentration using the adapted method for linear chloride build-up has an exact match to the finite difference method. Table 2 shows the sample calculations using equations (21), (22) and (23) for depth (x) = 20 mm.

Exponential chloride build-up

Table 3 presents the predicted chloride concentration at various depths for a concrete slab with $w/c = 0.4$. The maximum surface chloride (C_0) was equal to 1.0% by mass of concrete. It was assumed that the surface chloride build-up was exponential with $\alpha = 0.25$ and the first exposure to chloride occurred after 28 days. The diffusion coefficient ($D_{ref} = 7.94 \times 10^{12}$) was calculated using equation suggested in Life-365⁷ and m was taken as 0.3. Table 3 values were calculated after 20 years of exposure to chloride and five equal steps ($n = 5$) used in this calculation. It can be seen from Table 3 that the predicted results from the adapted method for exponential chloride build-up have close agreement with the finite difference method. Table 4 shows the sample calculations using equations (24), (25) and (23) for depth (x) = 20 mm.

Table 2. Sample calculations using equations (21), (22) and (23)

n	τ_i	t_i	$C(x, t)$
1	0.86	0.47	0.14
2	1.65	1.25	0.14
3	2.43	2.04	0.13
4	3.22	2.82	0.13
5	4.00	3.61	0.13
		Total	0.67

Table 3. Chloride concentration for exponential chloride build-up

x (mm)	Prediction by adapted method	Finite difference
10	0.81	0.80
20	0.63	0.63
30	0.48	0.47
40	0.35	0.35
50	0.24	0.24
60	0.16	0.17

Table 4. Sample calculations using equations (24), (25) and (23)

n	τ_i	t_i	$C(x, t)$
1	0.96	0.50	0.14
2	2.10	1.51	0.13
3	3.70	2.85	0.13
4	6.41	4.90	0.12
5	20.00	9.94	0.11
		Total	0.63

Conclusions

- (a) A general closed-form solution is presented for chloride concentration predictions in this paper for varying D but constant C_0 .
- (b) To incorporate the effect of varying C_0 , an adaptation of the closed-form solution is presented and this solution can be used without the use of finite difference method or special computer software to predict the chloride concentrations. This new model produces results comparable with the more complex finite difference method.
- (c) The analysis time is a concern in the finite difference solutions even though the results are fairly accurate. Life-365 dynamically increases the time steps during the analysis to decrease the analysis time. This problem is eliminated in the approaches presented in this paper.
- (d) Using software like a black box to predict the chloride concentrations does not give control and understanding of parameters to the user. The user has limited influence on the way the calculations are performed and cannot modify the model. The approach presented in this paper gives far more flexibility to the user.
- (e) Monte Carlo simulation in service life prediction and reliability estimation is typically a time-consuming process. Therefore, combining finite difference solution (for chloride concentration prediction) with Monte Carlo simulation is impossible in terms of time required to perform the analysis. In those cases, the approach presented in this paper gives a faster solution.

References

1. YUNOVICH M. and THOMPSON N. G. Corrosion of highway bridges: economic impact and control methodologies. *Concrete International*, 2003, **25**, No. 1, 52–57.
2. BENTUR A., DIAMOND S. and BERKE N. S. *Steel Corrosion in Concrete: Fundamentals and Civil Engineering Practice*. E & FN Spon, London, 1997.
3. RODRIGUEZ O. G. and HOOTON R. D. Influence of cracks on chloride ingress into concrete. *ACI Materials Journal*, 2003, **100**, No. 2, 120–126.
4. CRANK J. *The Mathematics of Diffusion*. 1st edn. Oxford University Press, Oxford, 1956.
5. BAMFORTH, P. B. The derivation of input data for modelling chloride ingress from eight-year UK coastal exposure trials. *Magazine of Concrete Research*, 1999, **51**, No. 2, 87–96.

6. KASSIR M. K. and GHOSN M. Chloride-induced corrosion of reinforced concrete bridge decks. *Cement and Concrete Research*, 2002, **32**, No. 1, 139–143.
7. THOMAS M. D. A. and BENTZ E. C. *Life-365: Computer Program for Predicting the Service Life and Life-Cycle Costs of Reinforced Concrete Exposed to Chlorides*. American Concrete Institute, Detroit, MI, USA, 2000.
8. SANDBERG P., PETERSSON K. and JORGENSEN O. Field studies of chloride transport into high performance concrete. In: *Performance of Concrete in a Marine Environment*, American Concrete Institute, Detroit, MI, USA, SP-163. 1996, pp. 233–254.
9. SHERMAN M. R., McDONALD D. B. and PFEIFER D. W. Durability aspects of precast, prestressed concrete, Part 2: chloride permeability study. *PCI Journal*, 1996, **41**, No. 4, 77–95.
10. BERKE N. S. and ROSENBERG A. Technical review of calcium nitrite corrosion inhibitor in concrete. *Transportation Research Record*, 1989, 1211, 19–27.
11. BODDY A., BENTZ E., THOMAS M. D. A. and HOOTON R. D. An overview and sensitivity study of a multi-mechanistic chloride transport model. *Cement and Concrete Research*, 1999, **29**, No. 6, 827–837.
12. COLLINS F. G. and GRACE W. R. Specifications and testing for corrosion durability of marine concrete: the Australian perspective. *Proceedings of Fourth CANMET/ACI International Conference*, Sydney, Australia, 1997, pp. 757–776.
13. MANGAT P. S. and MOLLOY B. T. Prediction of long term chloride concentration in concrete. *Materials and Structures*, 1994, **27**, No. 170, 338–346.

Discussion contributions on this paper should reach the editor by 1 February 2005